Nondiffracting periodically space-variant polarization beams with subwavelength gratings

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A class of propagation-invariant vector fields with uniform intensity and periodically space-varying polarization is presented. The beams were formed using computer-generated space-variant subwavelength gratings. A theoretical analysis as well as experimental results using gratings designed for CO_2 laser radiation at a wavelength of 10.6 μm is discussed. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1480477]

It is well known that spatially bound electromagnetic fields propagating in free space undergo diffractive spreading, resulting in changes of their intensity and phase profiles. If the field is infinite in extent, this is not necessarily the case, and the beam may be propagation invariant or what is commonly referred to as nondiffracting. Two well-known examples of such nondiffracting beams are scalar two-dimensional Bessel beams[1] and scalar one-dimensional cosine and sine beams.[2] Interest in propagation-invariant beams has not been limited to the scalar regime, and extensions to nondiffracting vector fields have been extensively studied theoretically.[3] However, experimental studies have been limited. Although approximately radially polarized propagation-invariant fields have been generated by interferometric techniques,[4] general vector fields with space-variant polarization are difficult to produce, thereby limiting possibilities for experimental studies.

Previously, we experimentally demonstrated the formation of beams with space-variant polarization using computer-generated subwavelength gratings.[5] When the period of the grating is much smaller than the incident wavelength only the zeroeth order is a propagating order and the grating structure results in effective birefringence. Therefore by controlling the local direction and period of the grating, any desired polarization can be obtained. Furthermore, we showed that space-variant polarization manipulation is accompanied by a geometrical phase modification closely related to the Pancharatnam–Berry phase[6] and that this phase modification affects the propagation of the resulting beam. We recently demonstrated that this phase can be utilized to form complex phase operators which we called Pancharatnam–Berry phase optical elements (PBOEs).[7]

In this letter we demonstrate the formation of a class of nondiffracting beams with periodically space-varying polarization using PBOEs based on computer-generated subwavelength gratings. By correctly determining the local period and direction of the grating, we control the polarization and phase of the resulting field to yield propagation-invariant beams with uniform intensity and continuous space-varying polarization. We present a theoretical analysis and show experimental results using subwavelength gratings designed for CO_2 laser radiation at a wavelength of 10.6 μm. Propagation-invariant vector fields could be useful for applications such as volume holography and metrology and for the study of ion diffraction from spatially varying polarized fields.

Subwavelength gratings lead to effective birefringence. Therefore it is convenient to describe them using Jones calculus. In this representation a space-invariant subwavelength grating with its grooves oriented in the y direction will be represented by the Jones matrix,

$$P=\begin{pmatrix} t_x & 0 \\ 0 & t_y e^{i\phi} \end{pmatrix} \tag{1}$$

where t_x, t_y, and ϕ are the period-dependent transmission coefficients and retardation of the grating, respectively. If the grating is space varying, it can be represented by the space-varying Jones matrix $T[(x,y)]=R^{-1}[\theta(x,y)]PR[\theta(x,y)]$, where $R(\theta)$ is a two-dimensional rotation matrix and $\theta(x,y)$ is the local orientation of the grating. For convenience we switch to the helicity base in which $|R\rangle$ and $|L\rangle$ are unit vectors representing right-hand and left-hand circular polarization, respectively. When represented in this base, $T(x,y)$ has explicit form of

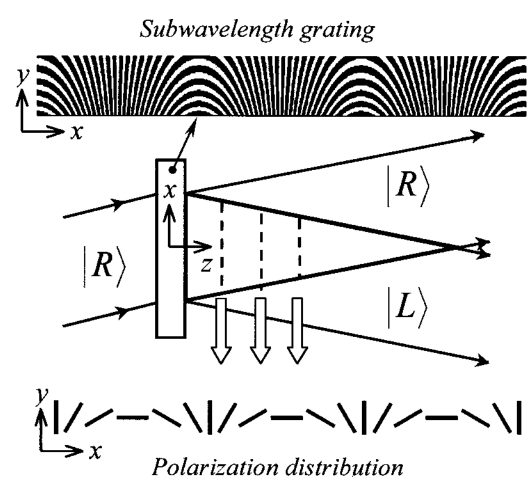

FIG. 1. Illustration of a PBOE for forming propagation-invariant fields. The insets show (top) the geometry of the PBOE, as well as (bottom) the vector field formed by it.

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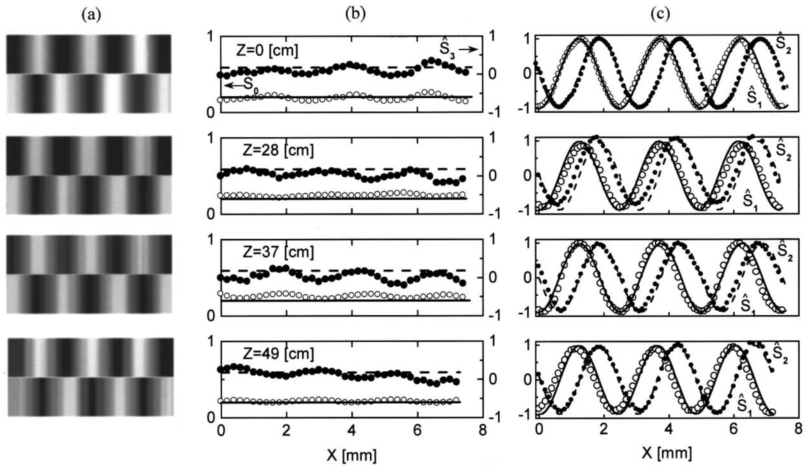

FIG. 2. (a) Measured intensity of the vertical and horizontal components of the beam as well as the normalized Stokes parameters, (b) S_3/S_0, and (c) S_1/S_0, S_2/S_0 at various planes along the axis of propagation. The solid and dashed lines indicate theoretical results whereas the dots indicate the experimental measurements.

$$T[\theta(x,y)]=\frac{(t_x+t_ye^{i\phi})}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}+\frac{(t_x-t_ye^{i\phi})}{2}\begin{pmatrix}0 & e^{i2\theta(x,y)}\\ e^{-i2\theta(x,y)} & 0\end{pmatrix}. \quad (2)$$

If we assume a space-varying periodic subwavelength grating with $\theta(x,y)=\pi x/d$ (d is the period), and illuminate the grating with an $|R\rangle$ polarized plane wave at an angle of incidence of $\sin\alpha=\lambda/2d$ (λ is the incident wavelength), then based on Eq. (2) the resulting field will be

$$|E_0(x,z=0)\rangle=T(x)e^{i(\pi x/d)}|R\rangle=\tfrac{1}{2}[(t_x+t_ye^{i\phi})e^{i(\pi x/d)}\times|R\rangle+(t_x-t_ye^{i\phi})e^{-i(\pi x/d)}\cdot|L\rangle]. \quad (3)$$

Figure 1 illustrates the result of Eq. (3). The transmitted beam comprises two orders, a zero order that maintains the original polarization, and does not undergo any phase modification and a diffracted order whose polarization has switched helicity, and whose phase is modified at each point. The two orders travel in different directions. We will show that interference of the two orders in the region where they overlap results in a propagation-invariant beam with constant intensity and transversely periodically varying polarization.

To prove that the beam is nondiffracting, we propagate it in the z direction using the Fresnel transfer function[8] $H=\exp[ikz(1-\lambda^2f^2/2)]$ (f is the spatial frequency and k is the wave number) to yield

$$|E_0(x,z)\rangle=\tfrac{1}{2}[(t_x+t_ye^{i\phi})e^{i(\pi x/d)}|R\rangle+(t_x-t_ye^{i\phi})e^{-i(\pi x/d)}|L\rangle]\times\exp[ikz(1-\lambda^2/8d^2)]. \quad (4)$$

Consequently we find that up to a constant phase $|E_0(x,z=0)\rangle=|E_0(x,z)\rangle$ for all z, and that the wave is nondiffracting. The propagation-invariant nature of $|E_0(x,z)\rangle$ can be better understood if we decompose the vector field into orthogonal linearly polarized components to achieve

$$E_x(x,z)=\frac{1}{\sqrt{2}}\left[t_x\cos\left(\frac{\pi x}{d}\right)+it_ye^{i\phi}\sin\left(\frac{\pi x}{d}\right)\right]\times\exp[ikz(1-\lambda^2/8d^2)], \quad (5a)$$

$$E_y(x,z)=\frac{1}{\sqrt{2}}\left[t_x\cos\left(\frac{\pi(x-d/2)}{d}\right)+it_ye^{i\phi}\sin\left(\frac{\pi(x-d/2)}{d}\right)\right]\times\exp[ikz(1-\lambda^2/8d^2)]. \quad (5b)$$

This reveals that $|E_0(x,z)\rangle$ is in fact the sum of two orthogonally polarized one-dimensional scalar nondiffracting beams. Furthermore we note that $E_y(x,z)=E_x(x-d/2,z)$. Since E_x and E_y are identical nondiffracting beams shifted relative to one another by half a period, their sum is also nondiffracting. Moreover, E_x and E_y are orthogonally polarized and therefore their interference does not result in intensity modulation, but rather in a wave front with uniform intensity and, as we will show, space-varying polarization.

To analyze the polarization of $|E_0(x,z)\rangle$ we calculate its Stokes parameters[9] as

$$S_1(x,z)=\tfrac{1}{2}(t_x^2-t_y^2)\cos(2\pi x/d)-t_xt_y\sin\phi\sin(2\pi x/d);$$

$$S_2(x,z)=\tfrac{1}{2}(t_x^2-t_y^2)\sin(2\pi x/d)+t_xt_y\sin\phi\cos(2\pi x/d);$$

$$S_3(x,z)=t_xt_y\cos\phi;\quad S_0(x,z)=\tfrac{1}{2}(t_x^2+t_y^2).$$

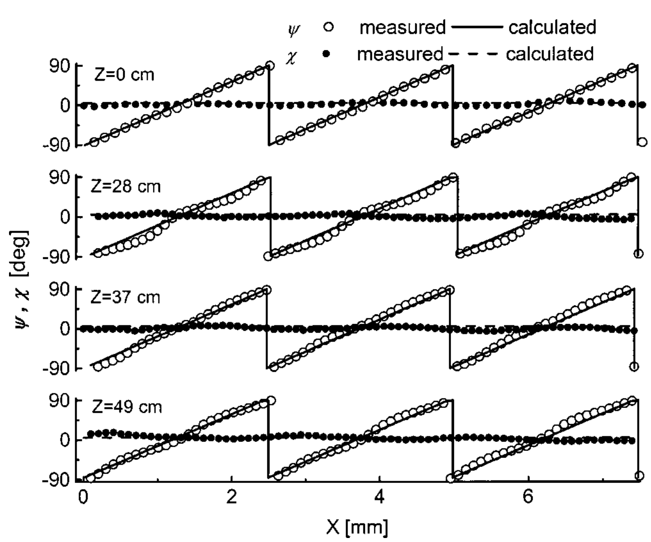

FIG. 3. Measured and predicted azimuthal angle and ellipticity of the nondiffracting beam at various planes.

Next we find the local ellipticity, χ, and the local azimuthal angle, ψ, of the beam from the connections,

$$\tan[2\psi(x,z)] = S_2/S_1 = \tan\left(2\pi x/d + \arctan\frac{2t_x t_y \sin\phi}{t_x^2 - t_y^2}\right), \tag{6a}$$

$$\sin[2\chi(x,z)] = S_3/S_0 = \frac{2t_x t_y \cos\phi}{t_z^2 + t_y^2}. \tag{6b}$$

Based on Eqs. (6) we find that the ellipticity of the beam is constant and depends on the transmission and retardation of the grating, whereas the local azimuthal angle varies periodically along the x direction at the same rate at which the local subwavelength grating's orientation varies, as depicted in Fig. 1. We thus conclude that any subwavelength grating whose local grooves are oriented at angle $\theta(x,y) = \pi x/d$ will produce a propagation-invariant field with uniform intensity and a nonuniform polarization structure when illuminated with an off-axis circularly polarized beam at an angle of $\sin\alpha = \lambda/2d$. This is true regardless of the transmission and retardation of the grating, i.e., regardless of the grating groove profile. We therefore turn our attention to a general design procedure for such gratings.

We begin by defining a grating vector $\mathbf{K}_g = K_0(x,y) \times [\cos(\pi x/d)\hat{\mathbf{x}} + \sin(\pi x/d)\hat{\mathbf{y}}]$ oriented perpendicular to the grating stripes.[10] $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are unit vectors in the x and y directions and $K_0 = 2\pi/\Lambda(x,y)$ is the spatial frequency of the grating (Λ is the local subwavelength period). To ensure continuity of the grating stripes we require $\nabla \times \mathbf{K}_g = 0$, from which $\Lambda(x,y)$ is determined. Finally, once $\mathbf{K}_g$ has been found, we calculate the grating function ϕ_g (defined so that $\nabla\phi_g = \mathbf{K}_g$) by integrating $\mathbf{K}_g$ over an arbitrary path yielding the solution

$$\phi_g(x,y) = (2d/\Lambda_0)\sin(\pi x/d)\exp(-\pi y/d),$$

where Λ_0 is the subwavelength period of the grating at $y = 0$.

We realized a metal stripe grating described by the grating function ϕ_g for CO_2 laser radiation at a wavelength of 10.6 μm using a Lee-type procedure.[10] The geometry of the grating is shown in Fig. 1. We chose $d = 2.5$ mm and the total length of the element was 30 mm so that it encompassed a total of 12 periods. We fabricated the grating with a width of 1.3 mm so that the subwavelength period, Λ, varied from 2 to 10.2 μm. The grating was realized on a 500 μm thick GaAs wafer with an antireflection coating on the back side. The grating was fabricated using contact photolithography and a lift-off technique. The metal stripes consisted of 60 nm of gold on an adhesion layer of 10 nm of titanium. Experimental measurement of the retardation and transmission of the grating in the region where the subwavelength period was below the Wood anomaly[10] yielded $t_x = 0.6$, $t_y = 0.2$, and $\phi = 0.6\pi$. These values are close to the theoretical predictions achieved using rigorous coupled wave analysis[11] for a binary gold-stripe grating with a period of 2.5 μm and a duty cycle of $q = 0.6$.

Following fabrication we illuminated the grating with off-axis circularly polarized light, which we focused using a cylindrical lens with an F number of 6.66. This enabled us to illuminate only the region of the grating where the subwavelength period was below the Wood anomaly. We then measured the Stokes parameters of the resulting beam at several planes along the optical axis.

Figure 2(a) shows experimental images of the beam when it passed through a polarizer oriented horizontally, and when it passed through a polarizer oriented vertically at planes $z = 0$ and $z = 28$, $z = 37$, and 49 cm. We note that the linearly polarized components resemble cosine beams, and that the vertically polarized component is shifted by half a period relative to the horizontally polarized component as predicted by Eqs. (5). Figure 2(b) shows the measured intensity, S_0, as well as the normalized Stokes parameter $\hat{S}_3 = S_3/S_0$. There is good agreement between theory and experiment. We note the constant intensity throughout the image space as well as the constant value of $\hat{S}_3$. Figure 2(c) shows the experimental and predicted results for $\hat{S}_1 = S_1/S_0$ and $\hat{S}_2 = S_2/S_0$. The sinusoidal shapes of $\hat{S}_1$ and $\hat{S}_2$ indicate the variation of the local azimuthal angle along the x axis as discussed above.

Figure 3 shows the experimental and theoretical local azimuthal angle, ψ, and ellipticity, χ, calculated from the data in Figs. 2(b) and 2(c). We note the constant ellipticity and the linearly varying azimuthal angle predicted by Eqs. (6). The results displayed in Figs. 2 and 3 clearly demonstrate the propagation invariance of the vector field.

Finally, to experimentally verify that any space-variant subwavelength grating described by $\phi_g(x,y)$ will yield a propagation-invariant beam, we performed a set of experiments with a GaAs dielectric subwavelength grating characterized by $t_x = 0.8$, $t_y = 0.8$, and $\phi = 65°$. The polarization measurements of the transmitted field revealed a nondiffracting beam with constant intensity and ellipticity of $\chi = 12.5°$. The azimuthal angle of the field varied linearly in a manner similar to that described for the beam in Fig. 3, thereby providing additional experimental evidence of the nature of the class of propagation invariance discussed in this letter.

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