# Continuous-phase elements can improve laser beam quality

## Ram Oron, Nir Davidson, and Asher A. Friesem

Department of Physics of Complex Systems, Weizmann Institute of Science, Rehovot 76100, Israel

#### Erez Hasman

Optical Engineering Group, Faculty of Mechanical Engineering, Technion-Israel Institute of Technology, Haifa 32000, Israel

### Received March 27, 2000

Siegman [Opt. Lett. 18, 675 (1993)] showed that binary-phase plates cannot improve laser beam quality. We demonstrate that continuous spiral phase elements can improve the quality of beams that originate from a laser operating with a pure high-order transverse mode. A theoretical analysis is presented, along with experimental results obtained with a  $CO_2$  laser. The results reveal that a nearly optimal Gaussian output beam can be obtained with only a small decrease in the output power. © 2000 Optical Society of America OCIS codes: 140.3300, 140.3430, 140.3570, 050.5080, 070.6110.

In a laser resonator that operates with multimodes, the beam quality is relatively poor. Generally, when a high-quality laser beam is required, one introduces an aperture into the resonator to obtain a fundamental mode of Gaussian shape. However, this concomitantly results in a significant decrease in output power. Several methods of operating a laser with a single high-order mode with higher output power than the fundamental mode have been demonstrated. These methods include the insertion of wire grids, 1 discontinuous phase elements, 2 or spiral phase elements<sup>3</sup> (SPE's) into the laser resonator. Analysis of beam quality by means of entropy<sup>4</sup> (as opposed to the more common  $M^2$  value<sup>5</sup>) showed that the entropy of a single high-order mode is equal to that of a single Gaussian mode. Thus, it is allowed thermodynamically to transform without losses a high-order mode into a Gaussian beam. In principle, such a transformation can be achieved with diffractive optical elements that transform both the amplitude and the phase of a wave front.<sup>6</sup> Here we demonstrate a practical and efficient method of transforming such a single high-order mode beam into a nearly Gaussian beam, leading to a laser with a higher output power than that of the same laser operating with the fundamental Gaussian mode. We apply our method to helical modes, but this method can be applied to other modes, albeit with less-efficient transformation.

We start by considering the field distribution of a TEM mode inside a laser resonator. In cylindrical coordinates, the field distribution, denoted  $E(r,\theta)$  for a helical TEM $_{pl^*}$  (nondegenerate Laguerre–Gaussian) mode, can be expressed as

$$\begin{split} E(r,\theta) &= E_0 \rho^{l/2} L_p^{\ l}(\rho) \text{exp}(-\rho/2) \text{exp}(-il\theta) \\ &= R_p^{\ l}(r) \text{exp}(-il\theta) \,, \end{split} \tag{1}$$

where r and  $\theta$  are the cylindrical coordinates;  $E_0$  is the magnitude of the field,  $\rho=2r^2/w^2$ , with w as the spot size of the Gaussian beam;  $L_p{}^l$  are the generalized Laguerre polynomials of order p and index l; and  $R_p{}^l(r)$  is an overall amplitude term that is a function of r. Usually, since modes of

opposite angular momentum have the same radial distribution  $(R^l = R^{-l})$ , they appear simultaneously, leading to a  $\mathrm{TEM}_{pl}$  (degenerate Laguerre–Gaussian) mode with  $E(r,\theta) = R_p{}^l(r)\mathrm{cos}(l\theta)$ . Note that, in general, the intensity distribution of the  $\mathrm{TEM}_{pl}{}^*$  mode will have a circularly symmetric annular shape, whereas that of the  $\mathrm{TEM}_{pl}$  will have lobes. Also, the common doughnut-shaped mode is not a helical mode but is composed of a superposition of two degenerate modes. <sup>7,8</sup>

The basic configuration of a laser resonator with a helical mode and an arrangement for transforming the helical output beam into a nearly Gaussian beam are shown schematically in Fig. 1. A helical TEM<sub>0l\*</sub> beam, with a field distribution given by Eq. (1), emerges from the laser into which a reflective SPE is inserted.<sup>3</sup> The beam is collimated by a cylindrical lens, and the beam's quality, according to the  $M^2$  value, is given by  $^7$  1 + l. In the optical mode converter the collimated beam first passes through a transmissive SPE, which introduces a phase of  $\exp(il\theta)$ , thereby modifying the helical-phase distribution into a uniform distribution, yielding  $E(r,\theta) = R_p^{\ l}(r)$ .

Numerical calculations based on Fourier transformation of the near field and the second-order moments reveal that the phase modification with the external SPE reduces the  $M^2$  value significantly, from 1 + l to  $\sqrt{1+l}$ . This result is in contrast with that obtained with a laser operating with degenerate modes, for which a correcting binary-phase plate can improve the far-field intensity distribution but not the beam quality ( $M^2$  value).<sup>10</sup> Moreover, the phase modification significantly changes the far-field intensity distribution, yielding a high central lobe and low ring-shaped sidelobes that contain only a small portion of the total power (e.g., 6% for a laser operating with the TEM<sub>01\*</sub> mode). Thus, by exploitation of a simple spatial filter (e.g., a circular aperture), it is possible to obtain a further significant improvement in  $M^2$ . Specifically, we can obtain a nearly Gaussian beam, with an  $M^2$ of nearly 1 (theoretically, a value of 1.036 for the TEM<sub>01\*</sub> mode), with only a small decrease in output

© 2000 Optical Society of America

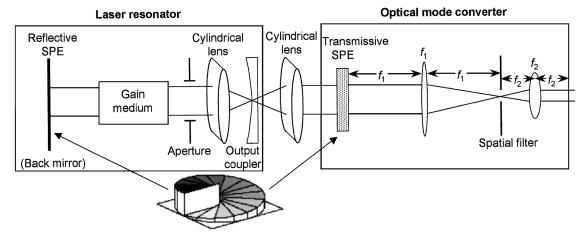


Fig. 1. Basic configuration of a laser resonator that yields a high-order helical mode and an optical mode converter that yields a nearly Gaussian mode.

power. Table 1 shows the calculated initial and final  $M^2$  values, as well as the transformation efficiency  $\eta$ , which denotes the percentage of power in the main lobe for a laser operating with various helical modes. Note that the transformation efficiency decreases as the order of the mode increases.

At this stage, we introduce a merit function, defined by  $B = P/M^2$ , where P is the power of the beam. With our optical mode converter, there should be an improvement in this merit function, in accordance with

$$\frac{B_{\rm final}}{B_{\rm initial}} = \frac{P_{\rm final}/M_{\rm final}^2}{P_{\rm initial}/M_{\rm initial}^2} = \eta M_{\rm initial}^2/M_{\rm final}^2 \,. \quad (2)$$

where  $B_{\rm final}$  and  $B_{\rm initial}$  denote the merit functions after and before the optical mode converter, respectively. Using Eq. (2), we calculated the improvement of the merit function versus the initial  $M^2$  value (given by 1+l) for different transverse helical modes. The results are presented in Fig. 2. As is evident from the figure, the improvement increases for the higher-order helical modes. For the corresponding degenerate modes, for which binary-phase modifications do not improve the  $M^2$  value, the improvement in the merit function is significantly smaller.

To evaluate our method experimentally we used the configuration shown in Fig. 1. The resonator was a linearly polarized CO<sub>2</sub> laser in which a reflective SPE (formed by 32-level reactive-ion etching of a silicon substrate) replaced the usual back mirror. We designed and formed the SPE to ensure that the laser operated in the helical TEM<sub>01\*</sub> mode. The length of the laser resonator was 65 cm, and the internal cylindrical lens, with a focal length of 12.5 cm, was focused on a concave output coupler (r = 3 m). A similar external cylindrical lens collimated the beam that emerged from the laser so it would be helical. The optical mode convertor contained a transmissive SPE formed on a zinc selenide substrate, a telescope configuration of two lenses, in which the first lens  $(f_1 = 50 \,\mathrm{cm})$  was placed 50 cm from the SPE and the second lens ( $f_2$  = 25 cm) was placed 75 cm from the first lens, and a spatial filter in the form of a circular aperture.

We detected the intensity distributions at the spatial-filter plane (25 cm in front of the second lens

in the optical mode converter) and the output plane (25 cm after the second lens) with a pyroelectric camera. The results are presented in Figs. 3 and 4. Figure 3 shows the detected intensity distributions, along with calculated and experimental cross sections at the spatial filter. The calculated results were obtained by Fourier transformation of the helical beam. Figure 3(a) shows the intensity distribution and cross sections without the transmissive SPE. Here we note the usual nearly doughnut-shaped distribution of a helical beam whose phase was not compensated for by the transmissive SPE. Figure 3(b) shows the intensity distribution and cross sections when the

Table 1. Initial and Final  $M^2$  Values and Transformation Efficiency  $\eta$  for a Laser Operating with Either the Fundamental Mode or High-Order Helical Modes

| Mode                                 | $TEM_{00}$ | $TEM_{01^*}$ | $TEM_{02^*}$ | $TEM_{03^*}$ | $TEM_{04^*}$ |
|--------------------------------------|------------|--------------|--------------|--------------|--------------|
| Initial $M^2$ Final $M^2$ $\eta$ (%) | 1          | 2            | 3            | 4            | 5            |
|                                      | 1          | 1.036        | 1.06         | 1.07         | 1.07         |
|                                      | 100        | 94           | 87           | 80           | 74           |

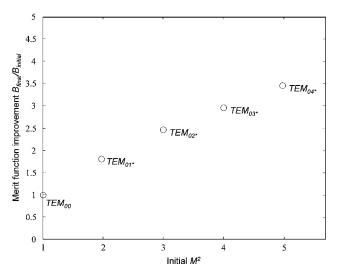


Fig. 2. Calculated improvement of merit function B versus the initial  $M^2$  value (given by 1+l) for different high-order helical modes.

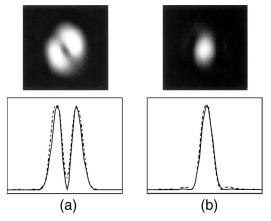


Fig. 3. Detected intensity distributions and (solid curves) calculated and (dashed curves) experimental cross sections at the spatial-filter plane (a) without and (b) with a transmissive SPE.

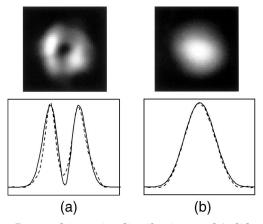


Fig. 4. Detected intensity distributions and (solid curves) calculated and (dashed curves) experimental cross sections at the output of the optical mode converter (a) without and (b) with a spatial filter.

transmissive SPE was inserted. As is evident, there is a high central peak, with low sidelobes that are removed by spatial filtering, yielding a nearly Gaussian beam. Moreover, the detected intensity distribution is narrower (by a factor of  $\sqrt{1+l}$ ) than that obtained with no SPE, indicating improvement of  $M^2$ .

Figure 4 shows photographs of the detected intensity distributions, along with calculated and experimental cross sections at the output of the optical mode converter. Here the calculated results were obtained by Fourier transformation of the field distribution in the spatial-filter plane. Figure 4(a) shows

the intensity distribution and cross sections at the output plane when the mode converter includes the SPE but no spatial filter. Here we simply image the doughnut-shaped helical beam from the laser, whose intensity distribution results from a TEM<sub>01\*</sub> mode. The SPE in this case affects not the intensity distribution at the output plane but only its phase. Figure 4(b) shows the detected intensity distribution and cross sections at the output plane with both the SPE and the spatial filter in the mode converter. As predicted, the intensity distribution has a Gaussian shape. In this case the efficiency  $\eta$  was 85%, which is somewhat lower than the calculated limit of 94%. The  $M^2$  value of this beam was measured to be better than 1.1, as expected, leading to an improvement of more than 1.5 in the merit function, in reasonable agreement with the predicted value shown in Fig. 2

To summarize, one can significantly improve the beam quality of beams originating from a laser operating with a single high-order transverse mode by exploiting continuous-phase elements. Moreover, one can obtain nearly optimal beam quality by simple filtering. We conclude that the  $M^2$  criterion, which is very useful for lasers operating in either the single fundamental mode or in multimodes, may be inappropriate for lasers operating with a single high-order transverse mode, whose phase distribution is well defined and can be modified. A more appropriate criterion could be the ratio of the output power to the modified  $M^2$ .

This research was supported in part by Pamot Venture Capital Fund and in part by the Eshkol Fund of the Israeli Ministry of Science. R. Oron's e-mail address is ram.oron@weizmann.ac.il.

## References

- K. M. Abramski, H. J. Baker, A. D. Colly, and D. R. Hall, Appl. Phys. Lett. 60, 2469 (1992).
- 2. R. Oron, Y. Danziger, N. Davidson, A. A. Friesem, and E. Hasman, Appl. Phys. Lett. **74**, 1373 (1999).
- 3. R. Oron, Y. Danziger, N. Davidson, A. A. Fiesem, and E. Hasman, Opt. Commun. 169, 115 (1999).
- 4. T. Graf and J. E. Balmer, Opt. Commun. 131, 77 (1996).
- 5. A. E. Siegman, Proc. SPIE 1224, 2 (1990).
- N. Davidson, A. A. Friesem, and E. Hasman, Appl. Phys. Lett. 61, 381 (1992).
- N. Hodgson and H. Weber, Optical Resonators (Springer-Verlag, Berlin, 1997), Chap. 5.2.
- A. E. Siegman, Lasers (University Science, Mill Valley, Calif., 1986), p. 689.
- 9. L. W. Casperson, N. K. Kincheloe, and O. M. Stafsudd, Opt. Commun. **21**, 1 (1977).
- 10. A. E. Siegman, Opt. Lett. 18, 675 (1993).