Formation of linearly polarized light with axial symmetry by use of space-variant subwavelength gratings

Avi Niv, Gabriel Biener, Vladimir Kleiner, and Erez Hasman

Optical Engineering Laboratory, Faculty of Mechanical Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel

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We present a novel method for forming linearly polarized axially symmetric beams with various polarization orders that is based on computer-generated space-variant subwavelength gratings. We introduce and experimentally demonstrate that our space-variant polarization state manipulations are accompanied by a phase modification of a helical structure that results from the Pancharatnam–Berry phase. We have verified the polarization properties of our gratings for laser radiation at 10.6-μm wavelength. © 2003 Optical Society of America

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Recent years have witnessed a growing interest in beams of a transversally space-variant polarization state. One of the most interesting types of such beams is the linearly polarized axially symmetric beam (LPASB). LPASBs are characterized by their polarization orientation, $\psi(\omega) = m\omega + \psi_0$, where m is the polarization order, ω is the azimuthal angle of the polar coordinate system, and ψ_0 is the initial polarization orientation for $\omega = 0$. Figure 1(a) illustrates LPASBs of polarization order $m = 1$ and $m = 2$. Note that a LPASB has a singular of its polarization state at the beam axis and, therefore, has a vectorial vortexlike structure. The most widely known members of the LPASB family are radial ($m = 1$, $\psi_0 = 0$) and azimuthal ($m = 1$, $\psi_0 = \pi/2$) beams, which are used extensively in applications such as particle acceleration, atom trapping, optical tweezers,[1] material processing, and tight focusing. Both radial and azimuthal beams can be formed by use of such methods as interferometric techniques and the intracavity summation of two orthogonally polarized TEM_{01} modes.[2] Recently the formation of higher polarization order LPASBs was demonstrated with liquid-crystal devices.[3,4] However, all these methods are somewhat cumbersome or unstable or have low efficiency.

We recently demonstrated the use of space-variant subwavelength gratings for the formation of radial and azimuthal beams.[5–7] In this Letter we present a novel design and fabrication procedure that allows computer-generated space-variant subwavelength dielectric gratings to be used to transform circularly polarized light into LPASBs that have, for the first time to our knowledge, any multiple of a half-integer polarization order. The continuity of our gratings ensures the continuity of the transmitted field, thus suppressing diffraction effects that may arise from discontinuity. Our space-variant polarization state manipulations are accompanied by a phase modification of a helical structure that results from the Pancharatnam–Berry phase. We also show that formation of a continuous half-integer polarization order LPASB must be accompanied by a Pancharatnam phase and that a beam with a Pancharatnam phase of any polarization order does not maintain its polarization state during propagation. We support our discussion with experimental results for CO_2 laser radiation at a wavelength of 10.6 μm. Finally, we demonstrate that a topological Pancharatnam charge can be defined for these beams and for its relation to the beams' angular momentum.

Subwavelength gratings have opened new methods for forming beams with sophisticated phase and polarization distributions. When the period of a grating is sufficiently smaller than the incident wavelength, only the zeroth order is a propagating order. Such gratings behave as layers of a uniaxial crystal.[8] Therefore, by using space-variant subwavelength gratings we can generate complex vectorial wave fronts with a different polarization state at each location. Two conditions have to be met if we wish to convert circularly polarized light into a LPASB by using subwavelength gratings. The first is conversion of the circularly polarized light into linearly polarized light by inducing $\pi/2$ retardation on the incident wave. The second is creation of the proper local polarization direction. Choosing the correct shape and form for the subwavelength grooves fulfills the first condition, whereas the second is met by creation of a local groove orientation of the form $\theta(\omega) = \psi(\omega) - \pi/4 = m\omega + \psi_0 - \pi/4$.

An axially symmetric space-variant subwavelength grating is typically described by a grating vector of the form $\mathbf{K}_g(r,\omega) = K_0(r,\omega)\{\cos[\theta(r,\omega) - \omega]\hat{r} + \sin[\theta(r,\omega) - \omega]\hat{\omega}\}$, where $\hat{r}$ and $\hat{\omega}$ are unit vectors in polar coordinates [Fig. 1(b)] and $K_0(r) = 2\pi/\Lambda(r,\omega)$ is the local spatial frequency for a grating of local period $\Lambda(r,\omega)$. Next, to ensure the continuity of the grating, we require that $\nabla \times \mathbf{K}_g = 0$, resulting in a differential equation that can be solved to yield the local grating period. The solution to this problem

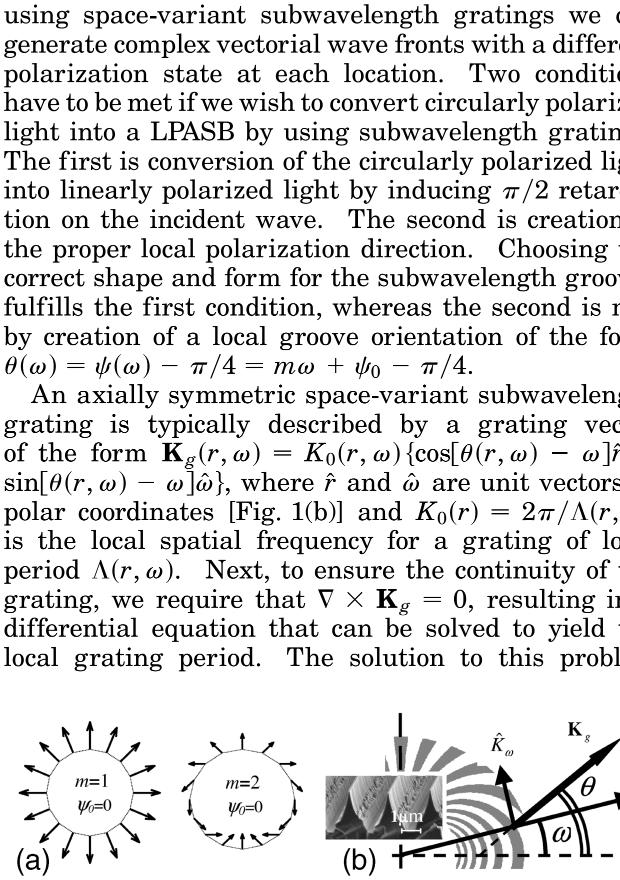

Fig. 1. (a) Illustration of linearly polarized light with axial symmetry and different polarization orders; (b) the geometrical definition of the grating vector. Inset, scanning electron microscope image of a typical cross section of the grating profile.

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yields $K_0(r) = (2\pi/\Lambda_0)(r_0/r)^m$, where Λ_0 is the local subwavelength period at $r = r_0$. Integrating $\mathbf{K}_g$ over an arbitrary path yields the desired grating function (defined such that $\nabla\phi_g = \mathbf{K}_g$) as

$$\phi_g(r,\omega) = 2\pi r_0 (r_0/r)^{m-1} \sin[(m-1)\omega + \psi_0 - 3\pi/4]/[(m-1)\Lambda_0], \qquad m \neq 1,$$

$$\phi_g(r,\omega) = (2\pi r_0/\Lambda_0)[\ln(r/r_0)\cos(\psi_0 - \pi/4) + \omega\sin(\psi_0 - \pi/4)], \qquad m = 1.$$

We then achieved Lee-type binary grating functions[9] for $m = 1/2, 1, 1\,1/2, 2$ and $\psi_0 = \pi/4$. The gratings were fabricated for CO_2 laser radiation with a wavelength of 10.6 μm with $\Lambda_0 = 2$ μm, $r_0 = 4.7$ mm, and a maximum radius of 6 mm. We formed the gratings with a maximum local period of 3.2 μm so we would not exceed the Wood anomaly of GaAs. A magnified geometry of the gratings is presented in Fig. 2 for several polarization orders. The elements were fabricated upon 500-μm-thick GaAs wafers by contact photolithography and electron-cyclotron resonance etching with BCl_3 to a nominal depth of 2.5 μm, which resulted in measured values of retardation $\phi = 0.4\pi$ and amplitude transmission coefficients of $t_x = 0.88$ and $t_y = 0.77$. These values are close to the theoretical predictions achieved by rigorous coupled-wave analysis. After the fabrication, an antireflection coating was applied to the backside of the element. The inset of Fig. 1 shows a scanning electron microscope image of a typical cross section of the grating profile.

Following the fabrication we illuminated the gratings with right-hand circularly polarized light at a wavelength of 10.6 μm from a CO_2 laser, imaged the emerging beam onto a Spiricon Pyrocam III infrared camera, and performed a full beam polarization characterization by measuring the Stokes parameters (S_0–S_3) at each point, using the four-measurements technique.[10] Figure 3(a) shows the first measurement, which is simply an image obtained by use of a linear polarizer as an analyzer. Note that for each beam the polarization state repeats itself $2m$ times. Figure 3(b) shows measured local azimuthal angle ψ of the resultant beams as predicted, indicating good control of the polarization direction when our method is used for various polarization orders. We found an average experimental deviation of ψ from the desired azimuthal angle of 0.3° and an average ellipticity $\tan(\chi)$ of 0.09, which we associate with a deviation of the retardation phase of the gratings from $\pi/2$ and with a fabrication error.

We gained additional insight by performing polarization and phase analyses of the elements. By representing the element as a space-variant Jones matrix, one can find the resultant wave front for any incident polarization.[6] For a space-varying quarter-wave plate and incident right-hand circular polarization, the Jones vector of the resultant beam is $\mathbf{E}_{\text{out}}(r,\omega) = [\cos(m\omega + \psi_0), \sin(m\omega + \psi_0)]^T \times \exp[-i(m\omega + \psi_0)]$. From here we can calculate the space-variant Pancharatnam phase[11] of the transmitted beam as $\varphi_P = \arg\langle\mathbf{E}(r,\omega), \mathbf{E}(r,0)\rangle$. In our case, the calculation of the Pancharatnam phase yields $\varphi_P = \arg[\cos(m\omega + \psi_0)] - m\omega$. This phase modification results solely from the polarization manipulation and is purely geometrical in nature.[12] The beam displays a Pancharatnam phase ramp of helical structure, similar to those found in scalar optical vortices; therefore we define the topological Pancharatnam charge of the beam as $l_P = (1/2\pi)\oint \nabla\varphi_P \mathrm{d}s = -m$. Note that in our case the Pancharatnam charge and the polarization order are equal in magnitude and opposite in sign. This charge can be modified by transmission of the beam through a spiral phase element of the form $\exp(il_d\omega)$ (l_d is an integer), in which case a topological charge of l_d is added to the beam.

Figures 4(a) and 4(c) show the calculated real parts of the instantaneous fields of LPASBs. Figure 4(a) shows the fields of the beams that are formed by

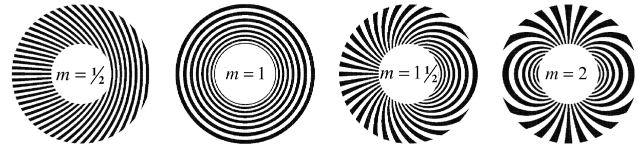

Fig. 2. Magnified geometry of the subwavelength gratings for polarization orders $m = 1/2, 1, 1\,1/2, 2$.

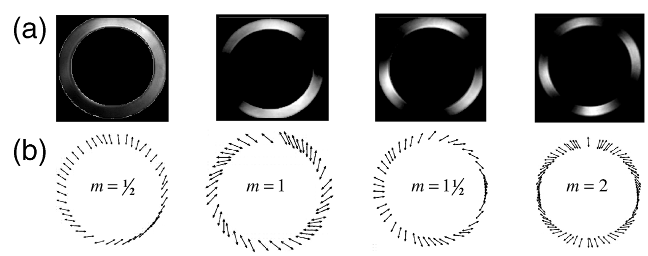

Fig. 3. (a) Experimental intensity distributions, directly after the gratings of different polarization orders, of the beams emerging from a linear polarizer as an analyzer; (b) measured local azimuthal angles of the beams.

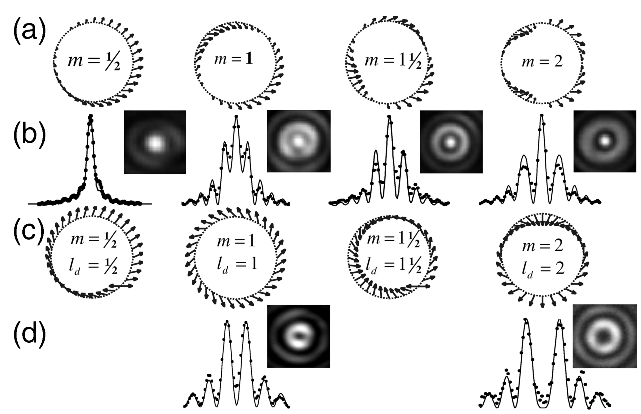

Fig. 4. Calculated real parts of the instantaneous vector fields for beams (a) emerging from the gratings only and (c) with the additional spiral phase element $l_d = m$. (b), (d) Experimental far-field images for the beams and their calculated (solid curves) and measured (filled circles) cross sections [(b) corresponds to (a), whereas (d) corresponds to (c)].

use of the gratings only for $m = 1/2, 1, 1 1/2, 2$ and $\psi_0 = \pi/4$, whereas Fig. 4(c) shows the beams that are created when, in addition to the grating, the waves of Fig. 4(a) are also transmitted through spiral phase elements that bear topological charge $l_d = m$, resulting in cancellation of the Pancharatnam phase while both the same space-variant polarization directions and the same polarization order m are maintained. Such beams are described by a Jones vector of the form $[\cos(m\omega + \psi_0), \sin(m\omega + \psi_0)]^T$. We define this polarization as being out of phase for which no phase modification has been introduced. When the polarization order is a half-integer (i.e., $m = 1/2, 1 1/2$), the fields of the beams that have a Pancharatnam phase [Fig. 4(a)] are continuous, whereas the fields of Fig. 4(c) without the Pancharatnam phase are discontinuous. It is evident that the formation of continuous LPASBs with half-integer polarization order is possible only for beams that have a topological Pancharatnam charge.

Although the corresponding beams of Fig. 4(a) and Fig. 4(c) of the same polarization order possess identical space-variant polarization directions, these consist of different phases, and it can be expected that they will propagate differently, producing distinguishable far-field images. Figure 4 shows also the experimental far-field images of these fields in their respective order [Fig. 4(b) corresponds to Fig. 4(a); Fig. 4(d) corresponds to Fig. 4(c)] as well as the calculated and measured cross sections. The beams transmitted through our subwavelength gratings of various polarization orders are converted into out-of-phase polarizations that have zero topological Pancharatnam charge by use of spiral phase elements formed by 32-level reactive-ion etching of ZnSe substrates. We note that the beams emerging through our gratings only [Fig. 4(b)] exhibit far-field images with bright centers, whereas the beams that undergo cancellation of the Pancharatnam phase exhibit distinct far-field images with dark centers [Fig. 4(d)]. There is a close connection between the instantaneous electric field [Figs. 4(a) and 4(c)] and the appearance of a bright or a dark spot at the far-field image of the beams. When the sum of the real part of the instantaneous electric field about the beam axis is zero, $\mathrm{Re}[\int_0^{2\pi} \mathbf{E}(\omega)\, \mathrm{d}\omega] = (0, 0)^T$, a dark spot at the far field is obtained and vice versa. The experimental results indicate that LPASBs with identical polarization orders and different Pancharatnam phases propagate in different manners, thereby emphasizing the relevance of correct phase determination in the propagation of space-variant polarization beams. We can easily explain the way in which the Pancharatnam phase influences the free propagation of the beam by expressing the beam as the coherent sum of two orthogonally polarized components with circular polarization. In the general case in which the spiral phase element is present, this decomposition yields

$$\mathbf{E}_{\mathrm{out}} = 1/2\big((1, -i)^T \exp(i l_d \omega) + (1, i)^T \exp\{-i[(2m - l_d)\omega + 2\psi_0]\}\big).$$

When $l_d = 0$ (no phase element) the beam consists of a scalar wave with a topological charge of $-2m$ and a wave with zero topological charge. The wave with charge $-2m$ conserves its vortex, whereas the wave with zero charge collapses and brings about the bright center. The process in which one scalar wave maintains its vortex while the other collapses changes the polarization state of the beam, and therefore LPASBs with Pancharatnam phase do not maintain their polarization state during propagation. When we apply the same logic to the waves that undergo a cancellation of the Pancharatnam phase ($l_d = m$), we find that the waves consist of two scalar vortices with topological charges m and $-m$. Neither of the two vortices collapses; consequently the dark spot is retained. Furthermore, the two scalar waves diffract in a similar manner and therefore LPASBs with zero Pancharatnam phase maintain their polarization state during propagation, as was experimentally verified for LPASBs of polarization orders $m = 1, 2$.

Another point of interest is the angular momentum of such beams. For a scalar wave, the angular momentum in the direction of propagation per unit energy is given by $\mathbf{J}_Z = (l + \sigma)/(2\pi v)$,[1] where l is the topological charge, σ is the helicity (± 1 for circular polarization), and v is the optical frequency of the beam. Using this rule and the decomposition of $\mathbf{E}_{\mathrm{out}}$ into circular polarization states yields the angular momentum of LPASBs as $\mathbf{J}_Z = 1/2 \sum_{i=L,R} (l_i + \sigma_i)/(2\pi v) = (l_d - m)/(2\pi v) = l_p/(2\pi v)$, where L and R indicate components with left and right circular polarization, respectively, and the angular momentum of these waves is given by the topological Pancharatnam charge. Our result introduces a connection between angular momentum and topological Pancharatnam charge.

E. Hasman's e-mail address is mehasman@tx.technion.ac.il.

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