Spatial Fourier-transform polarimetry using space-variant subwavelength metal-stripe polarizers

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A novel method for rapid polarization measurement is suggested. The method is based on a periodic space-variant polarizer that can be realized by use of subwavelength metal-stripe gratings. The Stokes parameters of the incident beam are determined by Fourier analysis of the space-variant intensity transmitted through the grating, thus permitting real-time polarization measurement. We discuss the design and realization of such polarizers and demonstrate our technique with polarization measurements of CO_2-laser radiation at a wavelength of 10.6 μm. © 2001 Optical Society of America

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Polarization measurements are used for a large range of applications, such as ellipsometry,[1] biosensing,[2] quantum computing,[3] and optical communications.[4] One commonly used method for polarimetry is to measure the time-dependent signal when the beam is transmitted through a rotating polarizer or quarter-wave plate.[5] By Fourier analysis of this signal, one can derive the Stokes parameters of the beam. However, this method is relatively slow, since it relies on a series of consecutive measurements. This poses difficulties in using the method for applications that require the measurement of dynamic changes in polarization, such as polarization-mode dispersion compensation in optical communications.

In this Letter we present the space domain analogy of the rotating polarizer method. Using subwavelength metal-stripe gratings, we are able to create a space-variant polarizer whose transmission axis varies periodically along the x axis. By performing a Fourier analysis of the transmitted intensity, we can determine the polarization of the incident beam in real time. We present a theoretical discussion of our method, discuss the design and realization of the space-variant polarizer, and provide experimental polarization measurements of CO_2-laser radiation at a wavelength of 10.6 μm.

The polarization state of light can be described as a Stokes vector $(S_0, S_1, S_2, S_3)^T$. S_0 is the intensity of the incident beam, and $S_1 \ldots S_3$ represent its polarization state. In the Stokes representation, a polarizer with complex amplitude transmission coefficients t_x and t_y is represented by the 4×4 Mueller matrix

$$P = \frac{1}{2}\begin{bmatrix} A & B & 0 & 0 \\ B & A & 0 & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -D & C \end{bmatrix}, \tag{1}$$

where $A = |t_x|^2 + |t_y|^2$, $B = |t_x|^2 - |t_y|^2$, $C = 2\,\mathrm{Re}(t_x t_y{}^*)$, and $D = 2\,\mathrm{Im}(t_x t_y{}^*)$. A polarizer rotated by an angle β is described by the matrix

$$M(\beta) = R(-\beta)PR(\beta), \tag{2}$$

where $R(\beta)$ is the Mueller matrix for a rotator.[5]

Usually, polarizers are space-invariant operators, and their transmission axis is constant. However, recent work suggested the possibility of polarization gratings in which the orientation of the transmission axis varies periodically.[6,7] It was suggested that such gratings could be used for polarization analysis in the far field. We intend to show that these gratings can be used for polarimetry by analysis of the transmitted intensity in the near field.

Figure 1 is a schematic representation of a period space-variant polarizer for which the transmission axis, β, varies linearly in the x direction, i.e., $\beta = ax$, where a is an arbitrary constant. Based on Eq. (2), the Mueller matrix that describes this polarizer is

$$M(x) = R(-ax)PR(ax). \tag{3}$$

Suppose that a monochromatic plane wave in an arbitrary state of polarization, $(S_0, S_1, S_2, S_3)^T$, is incident upon such a polarizer. In this case the polarization state of the transmitted beam will generally be space variant, and its Stokes vector can be found as $[S_0{}'(x), S_1{}'(x), S_2{}'(x), S_3{}'(x)]^T = M(x)\,(S_0, S_1, S_2, S_3)^T$. In particular, the intensity transmitted through the polarizer will be

$$S_0{}'(x) = \frac{1}{2}\{AS_0 + B[S_1\cos(2ax) + S_2\sin(2ax)]\}. \tag{4}$$

Equation (5) describes a truncated Fourier series whose coefficients depend on the Stokes parameters of

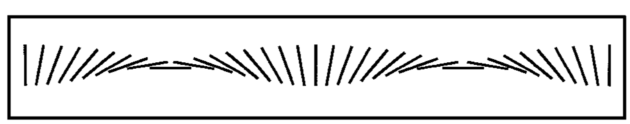

Fig. 1. Schematic representation of a space-variant periodic polarizer. The lines indicate the local transmission axis.

the incident beam. Therefore a simple Fourier analysis yields these parameters as

$$S_0 = \frac{a}{\pi A} \int_{x=0}^{2\pi/a} S_0{}'(x)\mathrm{d}x\,,$$

$$S_1 = \frac{2a}{\pi B} \int_{x=0}^{2\pi/a} S_0{}'(x)\cos(2ax)\mathrm{d}x,$$

$$S_2 = \frac{2a}{\pi B} \int_{x=0}^{2\pi/a} S_0{}'(x)\sin(2ax)\mathrm{d}x\,. \tag{5}$$

Furthermore, for polarized light, $S_3 = (S_0{}^2 - S_1{}^2 - S_2{}^2)^{1/2}$, which permits full analysis of the incident polarization.

Such a polarizer can be realized by use of subwavelength metal-stripe gratings. When the period of such a grating is considerably smaller than the wavelength, only light that is polarized perpendicular to the wires is transmitted, whereas light that is polarized parallel to the wires is reflected. Therefore, by accurately determining the direction and period of the grating at each point, we can obtain the desired continuous space-variant polarizer.[8] We now describe the design and realization of such gratings in brief.

First, we define a grating vector:

$$\mathbf{K}_g = K_0(x,y)[\cos(ax)\hat{\mathbf{x}} + \sin(ax)\hat{\mathbf{y}}]\,, \tag{6}$$

where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are unit vectors in the x and y direction, respectively; $K_0 = 2\pi/\Lambda(x,y)$ is the spatial frequency of the grating (Λ is the local period); and ax is the space-variant direction of the vector, defined so that it is perpendicular to the metal stripes at each point. Note that the grating vector is parallel to the local transmission axis of the grating.

Next, to ensure the continuity of the grating, we require that $\mathbf{K}_g$ be a conserving vector, i.e., that $\nabla \times \mathbf{K}_g = 0$. This results in a differential equation with the solution

$$\mathbf{K}_g = \frac{2\pi}{\Lambda_0} \exp(ay)[\cos(ax)\hat{\mathbf{x}} + \sin(ax)\hat{\mathbf{y}}]\,, \tag{7}$$

where Λ_0 is the period at $y = 0$. Using Eq. (7), we can calculate grating function ϕ (defined so that $\nabla\phi = \mathbf{K}_g$) by integrating $\mathbf{K}_g$ over an arbitrary path to obtain

$$\phi(x,y) = \frac{2\pi}{a\Lambda_0} \sin(ax)\exp(ay)\,. \tag{8}$$

Equation (8) and (9) show that the constraint on the continuity of the grating results in a varying period that depends on the y coordinate. Therefore the transmission coefficients of the polarizer also vary in the y direction, as they are period dependent.[9] However, as long as we perform the intensity measurement along lines parallel to the x axis of the grating, these transmission coefficients remain constant and Eqs. (3)–(5) remain valid. We can use the varying period of the grating to reduce statistical measurement errors by performing several measurements simultaneously.

Nevertheless, we need to determine the dependence of the Mueller matrix elements, A and B, on the grating period. For this purpose, we fabricated a 3 mm $\times$ 5 mm chirped grating with a linearly varying period in the x direction from 2 to 3.5 μm. First, a chrome mask was fabricated by high-resolution laser lithography. The mask was then transferred onto a 500-μm-thick GaAs wafer by use of photolithography, and the metal stripes, which consisted of 10 nm of Ti and 60 nm of Au, were realized by use of a lift-off technique. The duty cycle of the grating after fabrication was $q = 0.62$. Finally, we applied an antireflection coating to the backside of the element. We then illuminated the grating with light from a CO_2 laser at a wavelength of $\lambda = 10.6$ μm and measured the dependence of the transmission coefficients $|t_x|^2$ and $|t_y|^2$ on the period. Based on these results, we found A and B.

Figure 2 shows the experimental dependence of A and B on the local period, as well as the theoretical calculations for a binary metal grating with a duty cycle of $q = 0.62$. The calculations were performed with rigorous coupled-wave analysis,[10] and there is good agreement between theory and experiment. One can see that both A and B decline slowly as the period increases toward 3.24 μm, where the Wood anomaly[8] appears. In the proximity of the Wood anomaly, B is negative, and the grating is no longer an effective polarizer.

Based on these results, we realized a Lee-type binary grating describing the function of Eq. (8). The element consisted of a 5 mm $\times$ 1.5 mm rectangle with $a = -18°/\text{mm}$, $\Lambda_0 = 2$ μm, $-90° < \beta < 0°$, and $2\ \mu\text{m} < \Lambda < 3.24\ \mu\text{m}$, so as not to exceed the Wood anomaly. The fabrication was done with the same technique and parameters as those used for the chirped grating.

Figure 3 illustrates the geometry of this grating, as well as images that were captured with an optical microscope. We note the intricate design of the grating and the continuous metal stripes. We illuminated the space-variant polarizer of Fig. 3 with linearly polarized light at a wavelength of 10.6 μm and then varied the azimuthal angle, ψ, of the incident beam by use of a half-wave plate. The transmitted intensity was imaged through a lens and recorded by a Spiricon Pyrocam I pyroelectric camera.

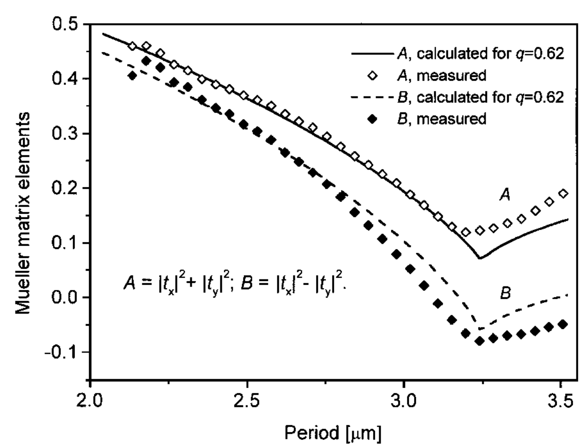

Fig. 2. Calculated and measured dependence of the Mueller matrix elements A and B on the period.

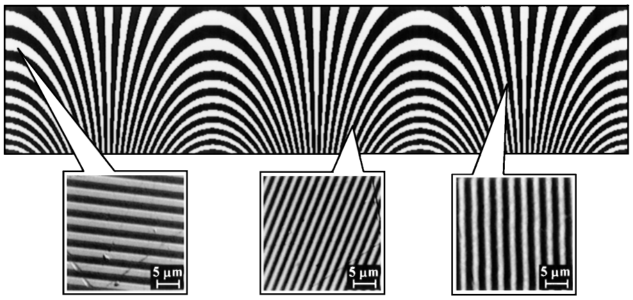

Fig. 3. Geometry of the subwavelength metal-stripe grating and images of the grating taken with an optical microscope.

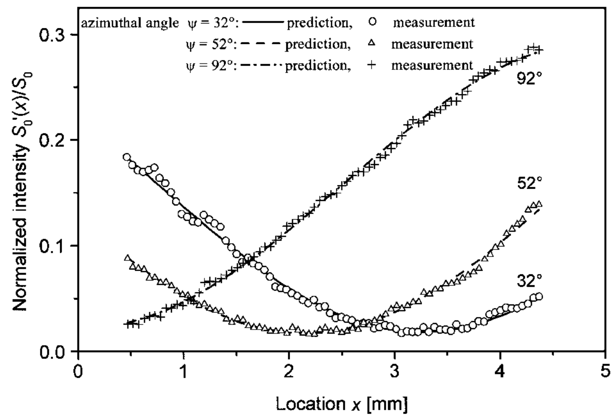

Fig. 4. Transmitted intensity as a function of the x coordinate at a local period of 2.13 μm when the polarizer is illuminated with linearly polarized light with three azimuthal angles.

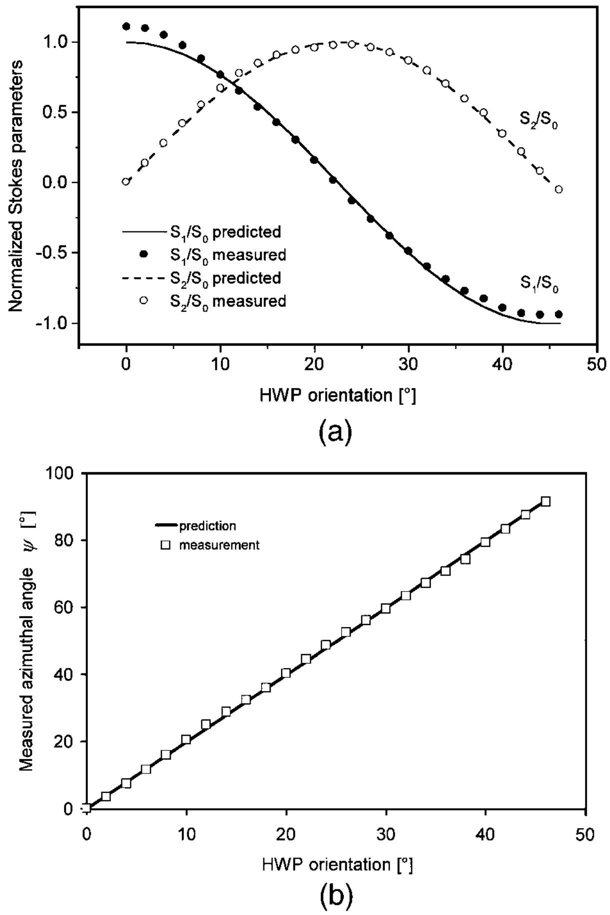

Fig. 5. Measured and predicted results of (a) Stokes parameters S_1 and S_2 and (b) azimuthal angle ψ as a function of the orientation of the half-wave plate (HWP).

Figure 4 shows the space-variant intensity distributions transmitted through the grating at a period of 2.13 μm for incident linearly polarized light with azimuthal angles of $\psi = 32°$, $\psi = 52°$, and $\psi = 92°$, along with the predicted results as calculated from Eq. (4) by use of the appropriate values of A and B from Fig. 2. There is good agreement between the predictions and the experimental results. The different incident polarizations produce distinct intensity distributions from which their Stokes parameters can be measured.

A series of such measurements is demonstrated in Fig. 5, which shows the experimental measurement of S_1 and S_2 and the azimuthal angle ψ of the incident beam as a function of the half-wave plate's orientation. We performed the measurement by fitting the curve of Eq. (4) to the measured intensities with S_1 and S_2 as free parameters. The azimuthal angle, ψ, was then calculated as $\tan(2\psi) = S_2/S_1$.[5] The standard deviation of the measured azimuthal angle from the predicted result is 0.6°, and there is a suitable fit between the measurements and the predicted results. We expect that, by use of a grating that consists of several cycles of the periodic function, the error can be reduced.

The same grating can be used to evaluate not only polarized light but partially polarized light as well. If we place a quarter-wave plate in front of the space-variant polarizer, then the transmitted intensity will be $S_0'(x) = ½\{AS_0 + B[S_1 \cos(2ax) - S_3 \sin(2ax)]\}$, thus permitting measurement of S_3. Such a wave plate can be realized as a dielectric subwavelength grating.[11]

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