Space-variant Pancharatnam–Berry phase optical elements with computer-generated subwavelength gratings

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Space-variant Pancharatnam–Berry phase optical elements based on computer-generated subwavelength gratings are presented. By continuously controlling the local orientation and period of the grating we can achieve any desired phase element. We present a theoretical analysis and experimentally demonstrate a Pancharatnam–Berry phase-based diffraction grating for laser radiation at a wavelength of 10.6 μm. © 2002 Optical Society of America

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The Pancharatnam–Berry phase is a geometric phase associated with the polarization of light. When the polarization of a beam traverses a closed loop on the Poincaré sphere, the final state differs from the initial state by a phase factor equal to half of the Ω area, encompassed by the loop on the sphere.[1,2] In a typical experiment, the polarization of a uniformly polarized beam is altered by a series of space-invariant (transversely homogeneous) wave plates and polarizers, and the phase that evolves in the time domain is measured by means of interference.[3,4]

Recently, we considered a Pancharatnam–Berry phase in the space domain. Using space-variant (transversely inhomogeneous) metal stripe subwavelength gratings, we demonstrated conversion of circular polarization into radial polarization[5] and showed that the conversion was accomplished by a space-variant phase modification of geometric origin that affected beam propagation.[6] Previously, Bhandari suggested the use of a discontinuous spatially varying wave plate as a lens based on similar geometric phase effects.[7] Recent studies have investigated periodic polarization gratings.[8–10] These authors showed that the polarization of diffracted orders could differ from polarization of the incident beam. We intend to prove and to utilize a connection between the properties of such polarization gratings and the space-domain Pancharatnam–Berry phase.

In this Letter we consider optical phase elements based on the space-domain Pancharatnam–Berry phase. Unlike diffractive and refractive elements, the phase is not introduced through optical path differences but results from the geometric phase that accompanies space-variant polarization manipulation. The elements are polarization dependent, thereby enabling multipurpose optical elements that are suitable for applications such as optical switching, optical interconnects, and beam splitting. We show that such elements can be realized using continuous computer-generated space-variant subwavelength dielectric gratings. The continuity of the gratings ensures the continuity of the resulting field, thereby eliminating diffraction associated with discontinuity and enabling the fabrication of elements with high diffraction efficiency. We experimentally demonstrate Pancharatnam–Berry phase diffraction gratings for CO_2 laser radiation at a wavelength of 10.6 μm, showing an ability to form complex polarization-dependent continuous-phase elements.

Figure 1 illustrates the concept of Pancharatnam–Berry phase optical elements (PBOEs) on the Poincaré sphere. Circularly polarized light is incident on a wave plate with constant retardation and a continuous space-varying fast axis whose orientation is denoted by $\theta(x, y)$. We show that, since the wave plate is space varying, the beam at different points traverses different paths on the Poincaré sphere, resulting in a space-variant phase-front modification that originates from the Pancharatnam–Berry phase. Our goal is to utilize this space-variant geometric phase to form novel optical elements.

It is convenient to describe PBOEs by use of Jones calculus. In this formalism, a wave plate with a space-varying fast axis is described by the operator

$$\mathbf{T}(x,y) = \mathbf{R}[\theta(x,y)]\mathbf{J}(\phi)\mathbf{R}^{-1}[\theta(x,y)],$$

where $\mathbf{J}(\phi)$ is the operator for a wave plate with retardation ϕ, $\mathbf{R}$ is the operator for an optical rotator, and θ is the local orientation of the axis at each point (x, y).

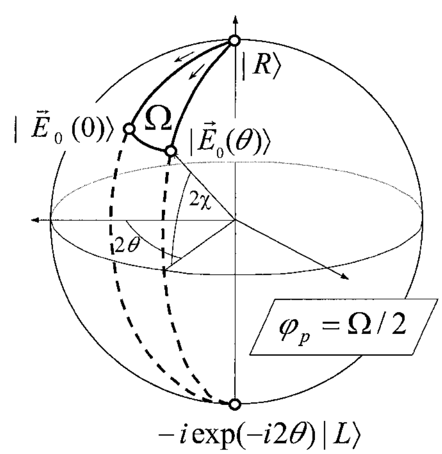

Fig. 1. Illustration of the principle of PBOEs by use of the Poincaré sphere.

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For simplicity, we work with the helicity basis in which $|L\rangle$ denotes left-hand circular polarization, and $|R\rangle$ denotes right-hand circular polarization. In this representation, $\mathbf{T}(x, y)$ has the explicit form

$$\mathbf{T}(x,y) = \cos(\phi/2)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - i\,\sin(\phi/2) \times \begin{Bmatrix} 0 & \exp[i2\theta(x,y)] \\ \exp[-i2\theta(x,y)] & 0 \end{Bmatrix} \cdot \quad (1)$$

Thus, if a beam with polarization $\mathbf{E}_i$ is incident on $\mathbf{T}(x, y)$, the resulting beam has the explicit form

$$\mathbf{E}_0 = \mathbf{T}(x,y)\mathbf{E}_i = \cos(\phi/2)\mathbf{E}_i - i\,\sin(\phi/2) \times [\langle \mathbf{E}_i|R\rangle \exp(-i2\theta)\,|L\rangle + \langle \mathbf{E}_i|L\rangle \exp(i2\theta)\,|R\rangle] \cdot \quad (2)$$

Using Eq. (2) we can calculate the Pancharatnam phase front of the resulting wave. We define this phase front based on Pancharatnam's definition for the phase between two beams of different polarizations[1] as $\varphi_p(x, y) = \arg[\langle \mathbf{E}_0(0, 0)|\mathbf{E}_0(x, y)\rangle]$. For incident $|R\rangle$ polarization this calculation yields

$$\varphi_p(x,y) = -\theta + \arctan[\cos\,\phi\,\tan\,\theta] = -\theta + \arctan[\sin(2\chi)\tan\,\theta],$$

where χ is the ellipticity of the resulting beam. Geometric calculations show that φ_p is equal to half of the area of geodesic triangle Ω on the Poincaré sphere defined by the pole $|R\rangle$, $|\mathbf{E}_0(0)\rangle$ and $|\mathbf{E}_0(\theta)\rangle$, as illustrated in Fig. 1, yield the expected Pancharatnam–Berry phase. Similar results can be found for any incident polarization.

Consequently, if a circularly polarized beam is incident on a space-variant polarization state manipulator, it is subject to geometric phase modification. Based on Eq. (2), the resulting wave consists of two components: the zero order and the diffracted order. The zero order has the same polarization as the original wave front and does not undergo any phase modification. On the other hand, the diffracted order has polarization orthogonal to that of the incoming wave, and its phase at each point is equal to twice the local orientation of wave plate $\theta(x, y)$. Since the phase modification of the wave front is purely geometric in origin, the phase of the diffracted orders must also be geometric. We therefore define the diffractive geometric phase (DGP) as the phase of the diffracted orders when the incident beam is circularly polarized. For incident $|R\rangle$ and $|L\rangle$ polarizations the DGP is equal to $-2\theta(x, y)$ and $2\theta(x, y)$, respectively. By correct determination of the local orientation of the wave plate, any desired DGP can be realized, enabling the realization of phase operators such as lenses or diffraction gratings. Furthermore, since the orientation of the wave plate varies only from 0 to π, the DGP is defined as modulus 2π, and the elements are analogous to diffractive optical elements. However, unlike diffractive optical elements the phase modification in PBOEs does not result from optical path differences, but from polarization state manipulation, and is intrinsically polarization dependent, with the transmission function given by the matrix $\mathbf{T}(x, y)$ as defined in Eq. (2). This equation also shows that the diffraction efficiency of the PBOEs depends on retardation of the wave plate.

A case of special interest is $\phi = \pi$, for which we found that the diffraction efficiency is 100% and that $|R\rangle$ polarization is completely converted into $|L\rangle$ polarization. However, despite the fact that the resulting polarization is space invariant, the Pancharatnam phase, $\varphi_p = -2\theta(x, y)$, is equal to the DGP. This phase corresponds to half of the area encompassed by two geodesic paths between the poles that form an angle of 2θ with one another, as illustrated in Fig. 1. Thus, the DGP is equal to the geometric Pancharatnam–Berry phase of a PBOE with 100% diffraction efficiency. Note that PBOEs operate in different ways on the two helical polarizations. Consequently, a PBOE lens designed for a wavelength λ, with focal length f, designed by choosing the direction of the wave plate so that $2\theta(x, y) = \pi r^2/\lambda f|_{\mathrm{mod}\ 2\pi}$ is a converging lens for $|R\rangle$ polarization and a diverging lens for $|L\rangle$ polarization.

PBOEs can be realized by use of space-variant subwavelength gratings. When the period of the grating is much smaller than the incident wavelength, the grating acts as a uniaxial crystal.[11] Therefore by correct control of the depth, the structure, and the orientation of the grating, the desired PBOE can be made. To design a PBOE, we need to ensure that the direction of the grating stripes, $\theta(x, y)$, is equal to half of the desired DGP, which we denote as $\varphi_d(x, y)$. Next we define a grating vector

$$\mathbf{K}_g = K_0(x,y)\{\cos[\varphi_d(x,y)/2]\hat{x} + \sin[\varphi_d(x,y)/2]\,\hat{y}\},$$

where $\hat{x}$ and $\hat{y}$ are unit vectors in the x and y directions, $K_0 = 2\pi/\Lambda(x, y)$ is the spatial frequency of the grating (Λ is the local subwavelength period), and $\varphi_d(x, y)/2$ is the space-variant direction of the vector defined so that it is perpendicular to the grating stripes at each point. Next, to ensure the continuity of the grating thereby ensuring the continuity of the resulting optical field, we require that $\nabla \times \mathbf{K}_g = 0$, resulting in a differential equation that can be solved to yield the local grating period. The grating function ϕ_g (defined so that $\nabla\phi_g = \mathbf{K}_g$) is then found by integrating $\mathbf{K}_g$ over an arbitrary path.[12]

We designed a PBOE that acts as a diffraction grating by requiring that $\varphi_d = (2\pi/d)x|_{\mathrm{mod}\ 2\pi}$, where d is the period of the structure. Applying this to the grating vector and solving the equation $\nabla \times \mathbf{K}_g = 0$ yields

$$\mathbf{K}_g = (2\pi/\Lambda_0)\exp(-\pi y/d)[\cos(\pi x/d)\hat{x} - \sin(\pi x/d)\hat{y}],$$

where Λ_0 is the subwavelength period at $y = 0$. The grating function is then found as $\phi_g(x, y) = (2d/\Lambda_0)\sin(\pi x/d)\exp(-\pi y/d)$. We then realized a Lee-type binary grating that describes the grating function ϕ_g.[13] The grating was fabricated for CO_2 laser radiation with a wavelength of 10.6 μm, with $\Lambda_0 = 2\ \mu$m and $d = 2.5$ mm, and consisted of

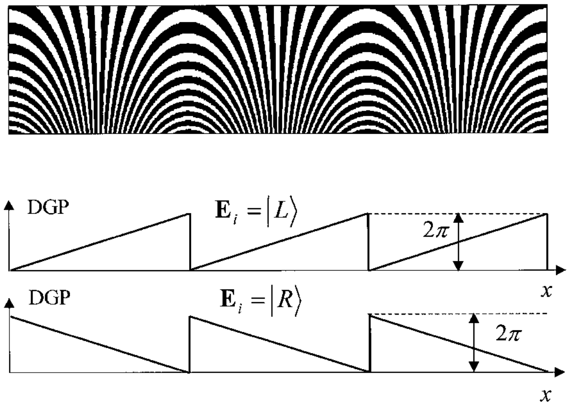

Fig. 2. Geometry of the space-variant subwavelength grating as well as the DGPs for incident $|R\rangle$ and $|L\rangle$ polarizations.

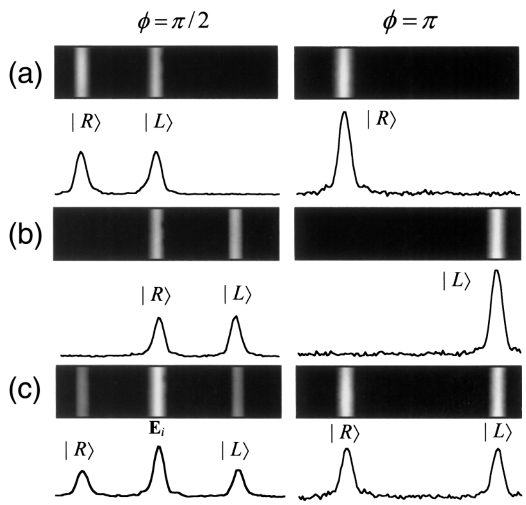

Fig. 3. Measurements of the transmitted far field for the subwavelength PBOE grating when the retardation is $\phi = \pi/2$ and $\phi = \pi$, respectively, for incident (a) circular left, (b) circular right, (c) linear polarizations.

12 periods of d. We formed the grating with a maximum local subwavelength period of $\Lambda = 3.2\ \mu$m, because the Wood anomaly occurs at 3.24 μm for GaAs.[11] We applied the grating to a 500-μm-thick GaAs wafer using contact printing and electron–cyclotron resonance etching with BCl_3 to a nominal depth of 2.5 μm to yield a retardation of $\phi = \pi/2$. By combining two such gratings we obtained a grating with $\phi = \pi$ retardation. Figure 2 illustrates the geometry of the grating, as well the DGP for incident $|L\rangle$ and $|R\rangle$ polarization states as calculated from Eq. (2). The DGPs resemble blazed gratings with opposite blazed directions for incident $|L\rangle$ and $|R\rangle$ polarization states, as expected from our previous discussions.

After fabrication, we illuminated the PBOEs with circular and linear polarization. Figure 3 shows the experimental images of the diffracted fields for the resulting beams, as well as their cross sections when the retardation was $\phi = \pi/2$ and $\phi = \pi$. When the incident polarization is circular and $\phi = \pi/2$, close to 50% of the light is diffracted according to a first-order DGP (the direction of diffraction depends on the incident polarization), whereas the other 50% remained undiffracted to zero order as expected from Eq. (2). The polarization of the diffracted order has a switched helicity as expected. For $\phi = \pi$, no energy appears in the zero order, and the diffraction efficiency is close to 100%. When the incident polarization is linear $\mathbf{E}_i = 1/\sqrt{2}(|R\rangle + |L\rangle)$, the two helical components of the beam are subject to different DGPs of opposite sign and are diffracted to first order in different directions. When $\phi = \pi/2$, the zero order maintains the original polarization, in agreement with Eq. (2), whereas for π retardation the diffraction is 100% efficient for both circular polarizations, and no energy is observable at zero order.

To conclude, we have demonstrated novel polarization-dependent optical elements based on the Pancharatnam–Berry phase. Unlike conventional elements, PBOEs are not based on optical path difference but on geometric phase modification resulting from space-variant polarization manipulation.

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