Dedicated to Professor IOAN URSU
on his 60th anniversary

OPTIMAL DESIGN FOR HOLOGRAPHIC FOCUSSING ELEMENTS

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This paper presents a method for designing optimal holographic optical elements. The optimization procedure is based on minimizing the mean-squared difference of the propagation vector components between the actual output wavefronts and the desired output wavefronts. Optimal solutions of the grating vectors for focussing elements are given.

INTRODUCTION

In an optical system, designed to operate with monochromatic illumination sources, it is possible to replace the conventional refractive elements with holographic optical elements (HOEs) [1]. Holographic elements have several advantages over conventional elements, they are thinner, more lightweight and can sometimes perform operations that are impossible by other means. Unfortunately, they have relatively large amount of aberrations. In order to minimize such aberrations, it is necessary to exploit optimization procedure for designing the grating function of the holographic element. Several procedures have been proposed. These are based on numerical iterative ray-tracing techniques [2] or on minimizing the mean-squared difference between the phases of the actual output wavefronts and those of the desired output wavefronts [3–6]. Such optimization procedures do not yield an exact solution except for very specific cases.

In this paper we present a different optimization procedure in which the design is based on analytic ray-tracing that minimizes the mean-squared difference of the propagation vector components between the actual output wavefronts and the desired output wavefronts. Thus, the optimal grating vector components of the holographic element can be solved analytically without any approximation. To illustrate our method we designed holographic focussing elements with, as well as without, stop apertures. The performance of the lenses was analyzed by ray-tracing and compared with conventional spherical HOEs.

THE OPTIMIZATION PROCEDURE

A holographic optical element can be described as a complex diffractive grating that transforms the phase of an incoming wavefront to another output phase. The phase of the output wavefront, $\psi_0(x, y)$, for the first diffracted order is given by

$$\psi_0(x, y) = \psi_i(x, y) - \psi_h(x, y). \tag{1}$$

where $\psi_i(x, y)$ is the phase of the input wavefront and $\psi_h(x, y)$ is the grating function of the HOE.

To proceed, we will now exploit the normalized propagation vectors and grating vector of the holographic element, rather than the phases. The normalized propagation vectors, which can be regarded as the direction cosines of the input ($\hat{\mathbf{K}}_i$) and output ($\hat{\mathbf{K}}_0$) rays, can be written as

$$\hat{\mathbf{K}}_0 = \frac{\lambda}{2\pi} \nabla\psi_0 \text{ and } \hat{\mathbf{K}}_i = \frac{\lambda}{2\pi} \nabla\psi_i, \tag{2}$$

and the grating vector $\mathbf{K}_h$, as

$$\mathbf{K}_h = \frac{\lambda}{2\pi} \nabla\psi_h = \frac{\lambda}{\Lambda_x} \hat{x} + \frac{\lambda}{\Lambda_y} \hat{y}, \tag{3}$$

where ∇ is the gradient operator, Λ_x and Λ_y are the grating spacing in x and y directions, and λ is the readout wavelength. The diffraction relation can now be written as

$$\hat{K}_{x_0} = \hat{K}_{x_i} - \hat{K}_{x_h}, \tag{4}$$

$$\hat{K}_{y_0} = \hat{K}_{y_i} - \hat{K}_{y_h}, \tag{5}$$

$$\hat{K}_{z_0} = \pm\sqrt{1 - \hat{K}_{x_0}^2 - \hat{K}_{y_0}^2}. \tag{6}$$

Note that $\hat{K}_{x_0}^2 + \hat{K}_{y_0}^2$ should be less than one so as not to obtain evanescent wavefronts. The goal when designing HOEs is to transfer input rays into corresponding output rays that will be optimized for a given range of input parameters. For a single specific input parameter it is relatively easy to form a HOE that will yield the exact desired output rays. However, for a range of input parameters, it is necessary to optimize the grating vector so as to minimize the difference between the actual and the desired output rays. The optimization is achieved by minimizing the mean-squared difference between these two sets of rays.

To simplify the presentation of our optimization method, we will describe the method in one dimensional notation. The mean-squared difference of the propagation vectors is defined as

$$E^2 = \int_{-D}^{D} \int_{a_1(x)}^{a_2(x)} [\hat{K}_{x_d}(x, a) - \hat{K}_{x_0}(x, a)]^2 \, \mathrm{d}a \, \mathrm{d}x, \tag{7}$$

where the direction cosines of the output and desired rays, $\hat{K}_{x_0}(x, a)$ and $\hat{K}_{x_d}(x, a)$, depend on some input parameter a, and x is the space coordinate on the HOE. The limits of integration, $a_1(x)$ and $a_2(x)$, represent the upper and lower values of the parameter of the input waves that intercept the HOE at a point x. For example, this input parameter could be the direction cosine of the incoming waves, or the location of the input point sources. The holographic element aperture is $2D$. Inserting Eq. (4) into Eq. (7) yields

$$E^2 = \int_{-D}^{D} \int_{a_1(x)}^{a_2(x)} [\hat{K}_{x_d}(x, a) - \hat{K}_{x_i}(x, a) + K_{x_h}(x)]^2 \, da \, dx. \tag{8}$$

The optimal grating vector component $K_{x_h}(x)$, can be determined by minimizing E^2. It is sufficient, however, to minimize a simpler integral that we denote as $e^2(x_0)$,

$$e^2(x_0) = \int_{a_1(x_0)}^{a_2(x_0)} [\hat{K}_{x_d}(x_0, a) - \hat{K}_{x_i}(x_0, a) + K_{x_h}(x_0)]^2 \, da, \tag{9}$$

where x_0 represents an arbitrary coordinate x. Differentiating $e^2(x_0)$ with respect to $K_{x_h}(x_0)$ and setting the result to zero, yields

$$\int_{a_1(x_0)}^{a_2(x_0)} [\hat{K}_{x_d}(x_0, a) - \hat{K}_{x_i}(x_0, a) + K_{x_h}(x_0)] \, da = 0. \tag{10}$$

Thus the optimal grating vector component will be

$$K_{x_h}(x) = \frac{-1}{(a_2(x) - a_1(x))} \int_{a_1(x)}^{a_2(x)} [\hat{K}_{x_d}(x, a) - \hat{K}_{x_i}(x, a)] \, da. \tag{11}$$

Now, the corresponding optimal grating function can be found by using Eq. (3), as

$$\psi_h(x) = \frac{2\pi}{\lambda} \int K_{x_h}(x) dx. \tag{12}$$

For an on-axis holographic element, having circular symmetry, the one-dimensional optimization procedure can be extended to two-dimension by simply letting

$$\psi_h(x, y) = \psi_h(r) = \frac{2\pi}{\lambda} \int K_{r_h}(r) dr, \tag{13}$$

where $r = \sqrt{x^2+y^2}$. For an off-axis HOE, where the off-axis angle is relatively low, it is possible to obtain an approximate solution by simply adding a linear term to the on-axis design [4, 5],

$$\psi_h(x, y) = [\psi_h(r)]_{\text{on-axis}} + \alpha_r x, \tag{14}$$

where $\alpha_r = \sin \theta_r$ and θ_r is the off-axis angle.

OPTIMAL HOLOGRAPHIC FOCUSSING ELEMENT

The operation of an on-axis focussing element is described with the aid of the one-dimensional representation in Fig. 1. It focuses each of the input plane waves to a point at the output plane corresponding to the angular direction of the input wave. Figure 1(a) shows a Holographic Focussing Element with a stop Aperture, (HFEA), and Figure 1(b) shows

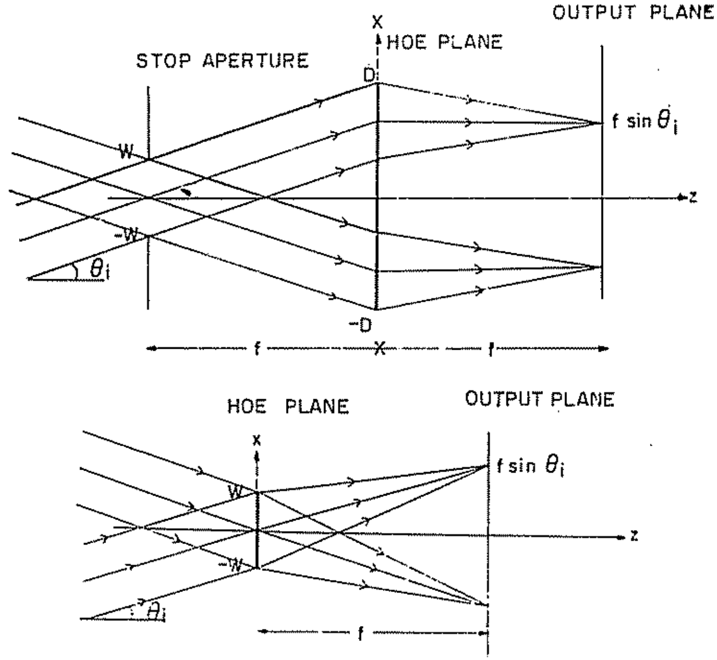

Fig 1. — The readout geometry of on-axis Holographic Focussing Element. (a) With stop aperture (HFEA); (b) Without stop aperture (HFE).

a Holographic Focussing Element without a stop aperture, (HFE). The HFEA has input stop aperture of $2W$, a holographic element aperture of $2D$, and it is centered along the optical axis Z. Finally, f is the distance from the holographic element to the stop aperture and to the output plane. The HFE has a holographic element aperture of $2W$, and focuses the incoming plane waves to a distance f.

It is convenient for focussing element design to let the input parameter a be the direction cosine of the input plane wave so

$$a = \alpha = \sin \theta_i. \tag{15}$$

Consequently, the normalized propagation vector of the input rays is

$$\hat{K}_{x_i}(x, a) = \hat{K}_{x_i}(\alpha) = \alpha. \tag{16}$$

Now, an input plane wave, having a direction cosine α, must be transformed into spherical wave converging to a point αf. Thus, the direction cosines of the desired output rays become

$$\hat{K}_{x_d}(x, a) = \hat{K}_{x_d}(x, \alpha) = \frac{-(x - \alpha f)}{\sqrt{(x - \alpha f)^2 + f^2}}. \tag{17}$$

Substituting $\hat{K}_{x_i}$ from Eq. (16) and $\hat{K}_{x_d}$ from Eq. (17) into Eq. (11), yields

$$K_{x_h}(x) = \frac{-1}{(\alpha_2(x) - \alpha_1(x))} \int\limits_{\alpha_1(x)}^{\alpha_2(x)} \left(\frac{-(x - \alpha f)}{\sqrt{(x - \alpha f)^2 + f^2}} - \alpha \right) \mathrm{d}\alpha, \tag{18}$$

where $\alpha_1(x)$ and $\alpha_2(x)$ are the lower and upper direction cosines of the input plane waves that intercept the holographic element at a point x.

The solution of Eq. (18) provides the final holographic grating vector as

$$K_{x_h}(x) = \frac{\alpha_1(x) + \alpha_2(x)}{2} - \frac{1}{(\alpha_2(x) - \alpha_1(x))} \left(\sqrt{\left(\frac{x}{f} - \alpha_2(x)\right)^2 + 1} - \sqrt{\left(\frac{x}{f} - \alpha_1(x)\right)^2 + 1} \right) \tag{19}$$

For the HFEA $\alpha_1(x)$ is given by

$$\alpha_1(x) = \frac{x - W}{\sqrt{(x - W)^2 + f^2}}, \tag{20a}$$

when $\alpha_1(x) > \dfrac{(-D + W)}{\sqrt{(-D + W)^2 + f^2}} = \alpha_{\min}$,

otherwise

$$\alpha_1(x) = \frac{(-D + W)}{\sqrt{(-D + W)^2 + f^2}}. \tag{20b}$$

$\alpha_2(x)$ is given by

$$\alpha_2(x) = \frac{x + W}{\sqrt{(x + W)^2 + f^2}}, \tag{21a}$$

when $\alpha_2(x) < \dfrac{(D - W)}{\sqrt{(D - W)^2 + f^2}} = \alpha_{\max}$, otherwise

$$\alpha_2 = \frac{(D - W)}{\sqrt{(D - W)^2 + f^2}}. \tag{21b}$$

For the HFE α_1 and α_2 are

$$\alpha_1 = \alpha_{min} = \sin\theta_{i\,min}, \tag{22a}$$

and

$$\alpha_2 = \alpha_{max} = \sin\theta_{i\,max}. \tag{22b}$$

As an illustration, we chose specific element parameters as $f = 60$mm, $W = 10$ mm, $D = 30$ mm, and the angular range of the input plane waves for as $\theta_{i\,max} = -\theta_{i\,min} = 18.4°$. We then designed a HFEA and a HFE according to our optimization procedure, and evaluated their performance using ray-tracing analysis [7]. For comparison we also evaluated the performance of a holographic spherical focussing element for which

$$[K_{x_h}(x)]_{sph} = \frac{x}{\sqrt{x^2 + f^2}}. \tag{23}$$

The criterion for evaluating the performance of the holographic elements is the size of the focussed points at the output plane. These spot sizes can be found by calculating the standard deviation of the location of the rays at the output plane as a function of the angular directions for each input plane wave.

The detailed results for the spot sizes as a function of the input angles were determined by using Eqs. (4), (16) and (19). The results, that do not take into account the diffraction from the apertures, are shown in Figs. 2 and 3. Figure 2 shows the results for the HFEA configuration. As shown, the spot sizes for the optimal HFEA are uniform over the entire range of input angles and they are significantly smaller than those for the spherical element. Figure 3 shows the results for the HFE configuration. As shown, the spot sizes for the optimal HFE are somewhat better than the spherical element. By comparing the results in Figs. 2 and 3, it is evident that the performance of the optimal HFEA is superior to that of the optimal HFE. The reason is that the design for the HFEA optimizes local holograms which the input plane waves intercept. Such localized optimization is obviously better than the simultaneous optimization of the entire HFE, where all input plane waves intercept a single area of the element. It should be emphasized, however, that the dimensions of the HFE are smaller than those of the HFEA.

We also calculated the amount of distortions by subtracting the actual (average) location of each spot from the desired location. Figure 4

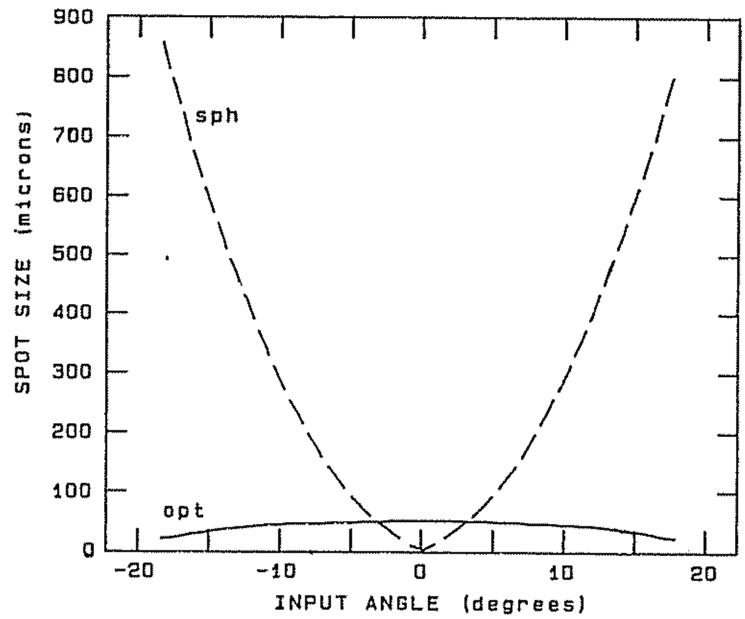

Fig. 2. — The spot size as a function of the input angle for HFEA; optimal (opt) and spherical (sph) grating functions.

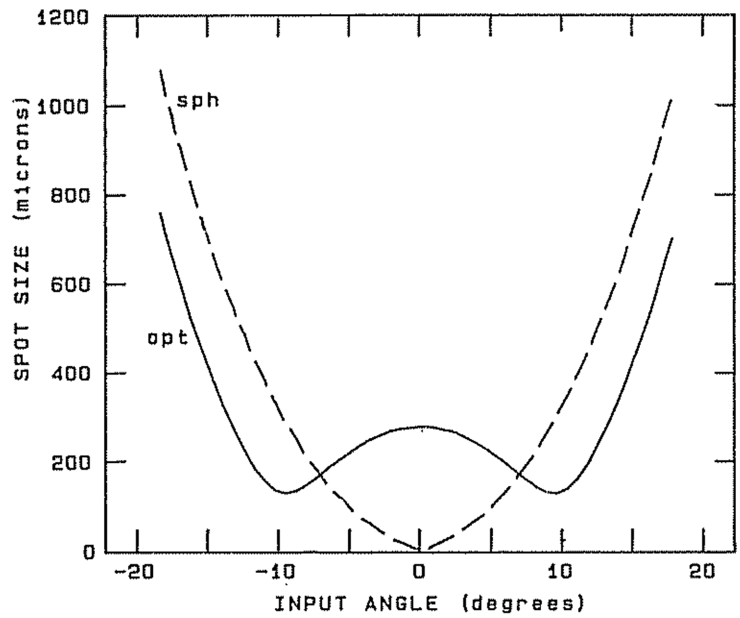

Fig. 3. — The spot size as a function of the input angle for HFE; optimal (opt) and spherical (sph) grating functions.

shows the distortions as a function of the input angle. The distortions for the optimal HFEA are significantly smaller than those for the HFE and the spherical elements.

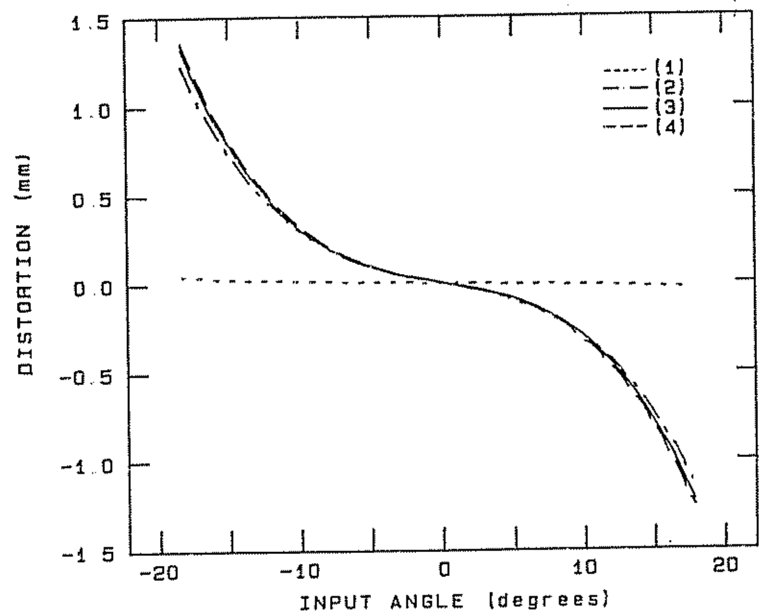

Fig. 4. — The distortion as a function of the input angle for optimal HFEA (1), spherical HFEA (2), optimal HFE (3), and spherical HFE (4) grating functions.

CONCLUDING REMARKS

We have presented a new optimization procedure for designing optimal HOEs which is based on analytic ray-tracing and relies on the normalized propagation vector components of the waves and the grating vector. To illustrate the optimization procedures, one-dimensional holographic focussing elements with and without stop apertures were designed and evaluated. The results revealed that optimally designed elements without stop aperture perform somewhat better than the conventional spherical holographic elements, while those having stop aperture are significantly better. This is due to the fact that in the latter case, at each hologram coordinate, the optimization is performed for a limited range of input angles rather than for the entire range as in the former case.

Finally, the grating function for an off-axis two-dimensional focussing element can be determined according to Eq. (14). Such a grating function can then be realized with computer generated or computer originated holograms.

We dedicate this article to Professor Ioan Ursu with our warmest wishes for his 60th birthday.

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