Blazed holographic gratings for polychromatic and multidirectional incidence light

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A new approach for optimizing the groove depth of blazed holographic gratings that are illuminated by light with a wide range of wavelengths or incidence angles is presented. The approach is based on choosing the groove depth that maximizes the overall diffraction efficiency over the entire range of incidence angles and wavelengths. The scalar approximation is used for the diffraction efficiency calculations, with some results verified by rigorous vectorial calculations. Analytic solutions are given for some simple examples, together with experimental results.

1. INTRODUCTION

Surface-relief diffraction gratings have attracted widespread interest because they can be exploited for a variety of applications. For example, these gratings serve as the diffraction elements in spectroscopy, as holographic lenses for both visible and other radiations, and as the elements for laser beam coupling. An important consideration for these surface-relief gratings is that their diffraction efficiencies be as high as possible. When the gratings are illuminated with a beam at a specific wavelength and a specific angle of incidence, it is possible to obtain 100% diffraction efficiency with a properly blazed groove shape. However, when the beam is polychromatic or is incident at orientation angles different from those for which the blazing was designed, the diffraction efficiency is reduced substantially.

Some attempts have been made to ensure high diffraction efficiency over a wide range of wavelengths, for example, by exploiting conical diffraction arrangements. We present a new approach for optimizing the groove depth of blazed gratings that are illuminated by light with a wide range of wavelengths or incidence angles. The approach is based on calculating the diffraction efficiency as a function of the incidence angle, the wavelength, and the groove depth and then choosing the depth that maximizes the overall diffraction efficiency over the entire range of angles and wavelengths. The diffraction efficiency is calculated by exploiting the scalar approximation, with some results verified by rigorous vectorial calculations. We present the general relations, analytic solutions for some simple examples, and experimental results.

2. BASIC RELATIONS

Blazed holographic gratings can be of either the transmissive or the reflective configurations. We chose to illustrate our approach with reflective gratings, but it can readily be applied to transmissive gratings as well. Figure 1 depicts the blazed reflection grating under consideration. A monochromatic plane wave with a wavelength \( \lambda \) is incident upon the grating at an angle \( \theta_i \). It is diffracted into several well-defined orders at angles \( \theta_m \) that are given by the diffraction relation

\[
\sin \theta_m - \sin \theta_i = m \lambda / \Lambda,
\]

where \( \Lambda \) is the grating period. We assume that the reflection from the grating is perfect, so we neglect the absorption. The depth profile of the blazed grating is expressed as

\[
f(x) = xd / \Lambda \quad \text{for } 0 < x < \Lambda,
\]

where \( d \) is the (maximal) groove depth. If \( \Lambda \) is much larger than \( \lambda \), the diffraction efficiency for any order can be obtained with the scalar approximation. Adapting the results for a dielectric grating to our reflection grating yields the following relation for the wave amplitude of the \( m \)th diffracted order \( R_m \):

\[
R_m = \frac{\exp(-i2\pi \Delta)}{2\pi \Delta}, \quad \text{where } \Delta = (d(\cos \theta_i + \cos \theta_m)/\lambda) - 1.
\]

The diffraction efficiency of the \( m \)th order, \( \eta_m \), is given simply by the square of the wave amplitude as

\[
\eta_m = R_m R_m^* = \frac{1}{2\pi^2 \Delta}(1 - \cos 2\pi \Delta),
\]

where \( \Delta \) is a function of the groove depth and the angle of incidence. This expression has a maximal value of 1 for \( \Delta = 0 \). Equation (6) indicates that, for an incident beam at a specific angle and...
The maximum of $E(d)$ can be found for any general distribution function $w(\theta_i, \lambda)$ by solving numerically the integral of Eq. (8) for different values of $d$. Such a numerical procedure involves lengthy and cumbersome calculations. Fortunately, there are some common and important applications with simpler distribution functions for which the optimal depth can be found analytically. We consider two specific examples, in which the distribution function has either one wavelength or one fixed angle.

3. RANGE OF INCIDENCE ANGLES

Let us consider a monochromatic incidence light whose angular distribution is uniform over the range $\theta_{\min} < \theta_i < \theta_{\max}$ and thus has the distribution function

$$w(\theta_i, \lambda) = \begin{cases} 0 & \text{if } \theta_{\min} < \theta_i < \theta_{\max} \\ 0 & \text{elsewhere} \end{cases}. \quad (9)$$

The overall diffraction efficiency of the grating is obtained by incorporating Eq. (9) into Eq. (8) to yield

$$E(d) = \int_{\theta_{\min}}^{\theta_{\max}} \eta_m(\theta_i, \lambda_0, d) d\theta. \quad (10)$$

To solve the integral of Eq. (10) analytically, we resort to several assumptions. First, we deal only with the first diffraction order ($m = 1$), so that, together with making the scalar grating assumption ($\Lambda \gg \lambda$), we may replace $\theta_m$ by $\theta_i$ in Eq. (5); actually, $\Lambda \geq 5\lambda$ is sufficient and results in an error of less that 1%. Second, we expand the cosine function in Eq. (6) in a power series and retain only the terms up to the fourth power; for angles of incidence of less than $45^\circ$ this approximation results in an error of less than 3%. Finally, we assume for simplicity that the range of incidence angles is symmetric about the normal, i.e., $\theta_{\min} = -\theta_{\max}$. Under these assumptions the integral of Eq. (10) may be solved analytically to obtain

$$E(d) = 1 - \frac{\pi^2}{6\theta_{\max}} [2\theta_{\max} - (2d/\lambda_0)4 \sin \theta_{\max} + (2d/\lambda_0)^2(\theta_{\max} + 0.5 \sin 2\theta_{\max})]. \quad (11)$$
Next, the wavelength of the CO₂ laser was changed to λ₀ = 9.6 µm, and the diffraction efficiency measurements were repeated. The maximal groove depth, still 5.3 µm, thus becomes 0.55λ₀ (instead of 0.5λ₀ in the previous experiment, where λ₀ was 10.6 µm). This corresponds to the optimal groove depth for the angular range of -45° < θ < 45°. The results are presented in Fig. 5, together with the corresponding theoretical results from Fig. 3. Again, the maximal experimental efficiency was normalized to 100% (here at an angle of incidence of 15°). By comparing Figs. 4 and 5, it is evident that the experimental diffraction efficiencies are indeed more uniform over the range of angles for the grating with the optimal groove depth of 0.55λ₀.

4. RANGE OF WAVELENGTHS

Let us now consider the relatively simple case in which the incoming light is oriented at a specific angle. We assume a normal incidence (θ = 0) and a uniform distribution of the spectral range λₘᵦ < λ < λₘₜ (white light). These assumptions are expressed mathematically by the distribution function having the form

\[
\omega(\theta, \lambda) = \begin{cases} 
\delta(\theta) & \text{if } \lambda_{\min} < \lambda < \lambda_{\max} \\
0 & \text{elsewhere}
\end{cases}
\]

The overall diffraction efficiency of the grating is obtained by incorporating Eq. (13) into Eq. (8) to yield

\[
E(d) = \int_{\lambda_{\min}}^{\lambda_{\max}} \eta_1(\theta = 0, \lambda, d) \, d\lambda.
\]

Following the same procedure as in Section 3 [i.e., Eqs. (10)-(12)], we find the optimal groove depth \(d_{\text{opt}}\) that maximizes the overall diffraction efficiency to be

\[
d_{\text{opt}} = 0.5 \frac{\lambda_{\max} \lambda_{\min}}{\lambda_{\max} - \lambda_{\min}} \ln \frac{\lambda_{\max}}{\lambda_{\min}}.
\]

It is interesting to compare this choice for the optimal groove depth with those that would give a diffraction efficiency of 100% for a specific wavelength within the range. In particular, we compare our choice with those where the

![Fig. 4. Experimental and calculated diffraction efficiencies as a function of the angle of incidence for a blazed grating with a groove depth of 0.5λ₀. Curve: scalar calculation; squares: experimental results.](image1)

![Fig. 5. Experimental and calculated diffraction efficiencies as a function of the angle of incidence for a blazed grating with a groove depth of 0.55λ₀ (optimal depth). Curve: scalar calculation; squares: experimental results.](image2)
tion efficiency with blazed diffraction gratings even when
We have shown that it is possible to obtain a high diffrac-
relatively moderate spectral ranges.
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5. CONCLUSION
are considerably different. It can therefore be concluded
depths derived in accordance with the arithmetic average
specific wavelength is an arithmetic average, \((\lambda_{\text{min}} +
\lambda_{\text{max}})/2\), and where it is a geometric average \((\lambda_{\text{min}}\lambda_{\text{max}})^{1/2}\).
We calculated the diffraction efficiencies and the optimal
groove depths for each choice. The results are shown in
Figs. 6 and 7. Figure 6 shows the diffraction efficiencies
as a function of \(\lambda\) (in units of \(\lambda_{\text{min}}\)) for a wavelength range
of \(\lambda_{\text{max}}/\lambda_{\text{min}} = 2\). As is shown, the diffraction efficiency
for the optimal groove depth is the most uniform over the
range of wavelengths.

Figure 7 shows the optimal groove depth (in units of \(\lambda_{\text{min}}\)) as a function of the ratio \(\lambda_{\text{max}}/\lambda_{\text{min}}\). As is evident,
the groove depths derived in accordance with the geometric
average are similar to the optimal depths. The groove
depths derived in accordance with the arithmetic average
are considerably different. It can therefore be concluded
that the simple geometric average may be adequate for
relatively moderate spectral ranges.

5. CONCLUSION
We have shown that it is possible to obtain a high diffrac-
tion efficiency with blazed diffraction gratings even when
the incident light contains a broad range of wavelengths
and arrives from a wide range of angles. The high effi-
ciencies are achieved by optimizing the groove depth in
the gratings. The gratings with the optimal groove depth
also have improved uniformity in diffraction efficiency
over the range of incidence angles and wavelengths, a
property that may be advantageous in many applications.
Finally, our optimization approach can be readily gen-
eralized to include higher diffraction orders and thick
gratings for which the period is of the same order of magni-
itude as the wavelengths. However, the scalar theory
may no longer be valid, so the diffraction efficiencies must
be calculated by the rigorous vectorial approach. In
such a rigorous approach the optimal groove depth could
be found by numerical rather than by analytic techniques.
Moreover, the small facet may no longer be vertical to the
grating plane, as for the thin grating, so its angle must be
optimized as well.

Throughout the paper we assumed that the groove shape
is triangular (blazed) and considered the effects of its
depth. It is possible that different groove shapes would
result in higher diffraction efficiencies over a broad range
of wavelengths and incidence angles. We considered bi-
nary, sinusoidal, and parabolic groove shapes, but these
resulted in lower diffraction efficiencies than were ob-
tained with the blazed gratings. Nevertheless, so far we
have not been able to prove rigorously that the blazed
groove shape is indeed optimal in this respect.

We conclude by noting that alternative approaches may
be considered when one deals with blazed holographic
gratings that are illuminated with nonlaser light. For
example, it is possible to require maximal uniformity of
the diffraction efficiency within the spectral and angular
range, rather than maximal total efficiency as we re-
quired. This would lead to some modifications in the
mathematical derivations, but the general formalism
would be similar.

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