Blazed holographic gratings for polychromatic and multidirectional incidence light

N. Davidson, R. Duer, A. A. Friesem, and E. Hasman

Department of Electronics, Weizmann Institute of Science, Rehovot 76100, Israel

Received October 28, 1991; revised manuscript received February 11, 1992; accepted February 12, 1992

A new approach for optimizing the groove depth of blazed holographic gratings that are illuminated by light with a wide range of wavelengths or incidence angles is presented. The approach is based on choosing the groove depth that maximizes the overall diffraction efficiency over the entire range of incidence angles and wavelengths. The scalar approximation is used for the diffraction efficiency calculations, with some results verified by rigorous vectorial calculations. Analytic solutions are given for some simple examples, together with experimental results.

1. INTRODUCTION

Surface-relief diffraction gratings have attracted widespread interest because they can be exploited for a variety of applications. For example, these gratings serve as the diffraction elements in spectroscopy, as holographic lenses for both visible and other radiations, and as the elements for laser beam coupling. An important consideration for these surface-relief gratings is that their diffraction efficiencies be as high as possible. When the gratings are illuminated with a beam at a specific wavelength and a specific angle of incidence, it is possible to obtain 100% diffraction efficiency with a properly blazed groove shape.[1-3] However, when the beam is polychromatic or is incident at orientation angles different from those for which the blazing was designed, the diffraction efficiency is reduced substantially.[4,5] Some attempts have been made to ensure high diffraction efficiency over a wide range of wavelengths, for example, by exploiting conical diffraction arrangements.[6]

We present a new approach for optimizing the groove depth of blazed gratings that are illuminated by light with a wide range of wavelengths or incidence angles. The approach is based on calculating the diffraction efficiency as a function of the incidence angle, the wavelength, and the groove depth and then choosing the depth that maximizes the overall diffraction efficiency over the entire range of angles and wavelengths. The diffraction efficiency is calculated by exploiting the scalar approximation,[1,7] with some results verified by rigorous vectorial calculations.[8] We present the general relations, analytic solutions for some simple examples, and experimental results.

2. BASIC RELATIONS

Blazed holographic gratings can be of either the transmittive or the reflective configurations. We chose to illustrate our approach with reflective gratings, but it can readily be applied to transmittive gratings as well. Figure 1 depicts the blazed reflection grating under consideration. A monochromatic plane wave with a wavelength λ is incident upon the grating at an angle θ_i. It is diffracted into several well-defined orders at angles θ_m that are given by the diffraction relation

$$\sin\theta_m - \sin\theta_i = m\lambda/\Lambda, \tag{1}$$

where Λ is the grating period. We assume that the reflection from the grating is perfect, so we neglect the absorption. The depth profile of the blazed grating is expressed as

$$f(x) = xd/\Lambda \qquad \text{for } 0 < x < \Lambda, \tag{2}$$

where d is the (maximal) groove depth. If Λ is much larger than λ, the diffraction efficiency for any order can be obtained with the scalar approximation.[1] Adapting the results for a dielectric grating[7] to our reflection grating yields the following relation for the wave amplitude of the mth diffracted order R_m:

$$R_m = \frac{1}{\Lambda}\int_0^\Lambda \exp[-imKx - ik(\cos\theta_i + \cos\theta_m)f(x)]\mathrm{d}x, \tag{3}$$

where $k = 2\pi/\lambda$ and $K = 2\pi/\Lambda$.

For gratings with a blazed groove shape, having the depth profile given by Eq. (2), the integral of Eq. (3) can be solved analytically to yield

$$R_m = \frac{i}{2\pi\Delta}[\exp(-i2\pi\Delta) - 1], \tag{4}$$

where

$$\Delta = [d(\cos\theta_i + \cos\theta_m)/\lambda] - 1. \tag{5}$$

The diffraction efficiency of the mth order, η_m, is given simply by the square of the wave amplitude as

$$\eta_m = R_m R_m{}^* = \frac{1}{2\pi^2\Delta^2}(1 - \cos 2\pi\Delta), \tag{6}$$

where $*$ denotes a complex conjugate. This expression has a maximal value of 1 for $\Delta = 0$. Equation (6) indicates that, for an incident beam at a specific angle and

0740-3232/92/071196-04$05.00

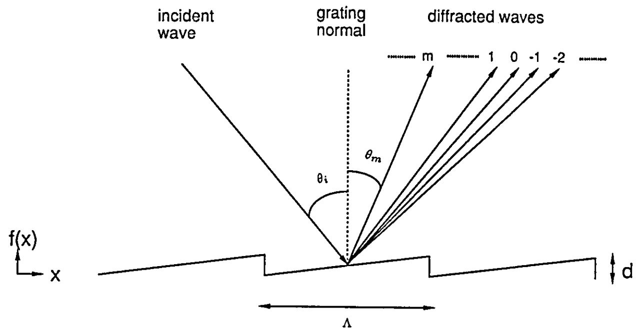

Fig. 1. Geometry of a blazed reflective holographic grating.

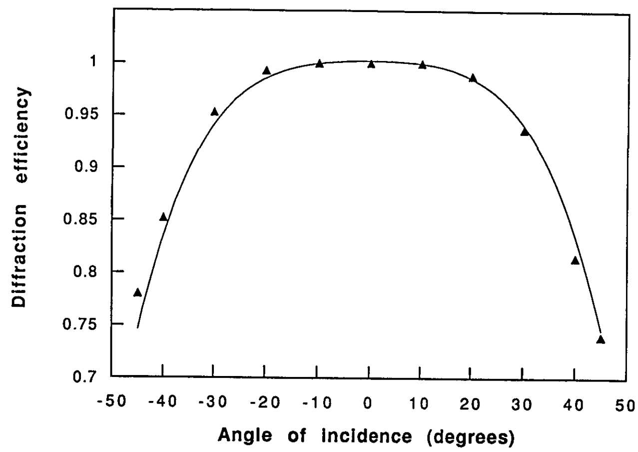

Fig. 2. Calculated diffraction efficiency as a function of the angle of incidence for a blazed grating with a groove depth of 0.50λ. Curve: scalar calculation; triangles: vectorial calculation.

wavelength, it is possible to obtain a diffraction efficiency of 100% with a maximal groove depth of

$$d = \frac{\lambda}{\cos\theta_i + \cos\theta_m}. \tag{7}$$

However, at other angles and wavelengths, the diffraction efficiency is reduced considerably. As an illustration, we calculated by using Eq. (6) the diffraction efficiency for incidence angles ranging from $-45°$ to $45°$. The grating period was $\Lambda = 30\lambda$, and the groove depth was $d = \lambda/2$ (appropriate for normal incidence). The results are shown in Fig. 2, together with the results obtained from a rigorous vectorial formalism[8] in which TE polarization was assumed. As is shown, good agreement exists between the scalar and the vectorial results, and both predict a decrease of as much as 25% in the diffraction efficiency at the extreme angles.

To find the optimal groove depth that would maximize the diffraction efficiency of the mth order for the entire range of angles and wavelengths, we propose to maximize the following expression for the overall diffraction efficiency:

$$E(d) = \iint \omega(\theta_i, \lambda)\eta_m(\theta_i, \lambda, d)\mathrm{d}\theta\mathrm{d}\lambda, \tag{8}$$

where $\omega(\theta_i, \lambda)$ is the angular and spectral energy distribution of the incident light and $\eta_m(\theta_i, \lambda, d)$ is given by Eq. (6). Note that $\omega(\theta_i, \lambda)$ may also account for practical considerations, such as the possible sensitivity variation of the detector. The maximum of $E(d)$ can be found for any general distribution function $\omega(\theta_i, \lambda)$ by solving numerically the integral of Eq. (8) for different values of d. Such a numerical procedure involves lengthy and cumbersome calculations. Fortunately, there are some common and important applications with simpler distribution functions for which the optimal depth can be found analytically. We consider two specific examples, in which the distribution function has either one wavelength or one fixed angle.

3. RANGE OF INCIDENCE ANGLES

Let us consider a monochromatic incidence light whose angular distribution is uniform over the range $\theta_{min} < \theta_i < \theta_{\max}$ and thus has the distribution function

$$\omega(\theta, \lambda) = \begin{cases} \delta(\lambda - \lambda_0) & \text{if } \theta_{\min} < \theta_i < \theta_{\max} \\ 0 & \text{elsewhere} \end{cases}. \tag{9}$$

The overall diffraction efficiency of the grating is obtained by incorporating Eq. (9) into Eq. (8) to yield

$$E(d) = \int_{\theta_{\min}}^{\theta_{\max}} \eta_m(\theta_i, \lambda_0, d)\mathrm{d}\theta. \tag{10}$$

To solve the integral of Eq. (10) analytically, we resort to several assumptions. First, we deal only with the first diffraction order ($m = 1$), so that, together with making the scalar grating assumption ($\Lambda \gg \lambda$), we may replace θ_m by θ_i in Eq. (5); actually, $\Lambda \geq 5\lambda$ is sufficient and results in an error of less that 1%. Second, we expand the cosine function in Eq. (6) in a power series and retain only the terms up to the fourth power; for angles of incidence of less than 45° this approximation results in an error of less than 3%. Finally, we assume for simplicity that the range of incidence angles is symmetric about the normal, i.e., $\theta_{\min} = -\theta_{\max}$. Under these assumptions the integral of Eq. (10) may be solved analytically to obtain

$$\begin{aligned} E(d) = 1 - \frac{\pi^2}{6\theta_{\max}}[&2\theta_{\max} - (2d/\lambda_0)4\sin\theta_{\max} \\ &+ (2d/\lambda_0)^2(\theta_{\max} + 0.5\sin 2\theta_{\max})]. \end{aligned} \tag{11}$$

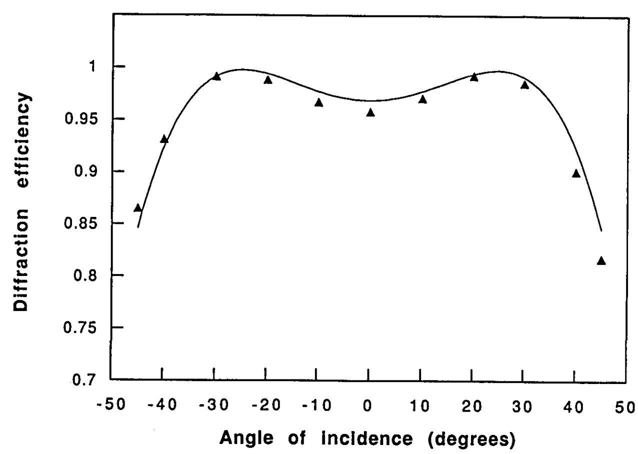

Fig. 3. Calculated diffraction efficiency as a function of the angle of incidence for a blazed grating with a groove depth of 0.55λ (optimal depth). Curve: scalar calculation, triangles: vectorial calculation.

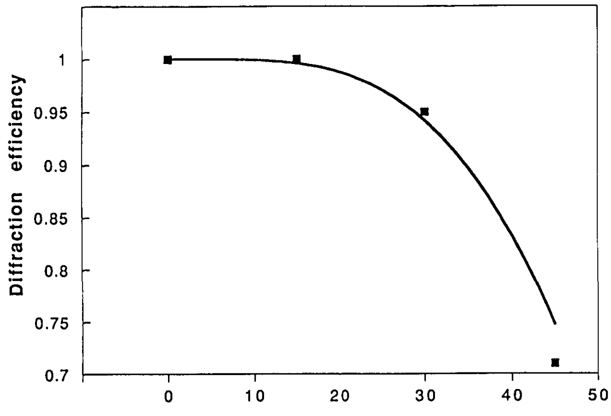

Fig. 4. Experimental and calculated diffraction efficiencies as a function of the angle of incidence for a blazed grating with a groove depth of 0.50λ. Curve: scalar calculation; squares: experimental results.

Now, to maximize $E(d)$ we require that the derivative of $E(d)$ with respect to d be set to zero, $\partial E(d)/\partial d = 0$. This yields the optimal groove depth d_{opt},

$$d_{\mathrm{opt}} = \frac{\lambda_0}{2} \frac{4 \sin \theta_{\max}}{2\theta_{\max} + \sin 2\theta_{\max}}. \tag{12}$$

For example, in accordance with Eq. (12), the optimal groove depth for the range $-45° < \theta < 45°$ is $d_{\mathrm{opt}} = 0.55\lambda_0$; this value for d_{opt} is 10% larger than the optimal groove depth for normal incidence ($d_{\mathrm{opt}} = \lambda_0/2$). The diffraction efficiency as a function of the angle of incidence was calculated by using Eq. (6) for a grating having the optimal groove depth. The results are shown in Fig. 3, again along with the results obtained by a rigorous vectorial formalism; all the parameters besides the groove depth were the same as those for Fig. 2. By comparing Figs. 2 and 3 it is evident that, for the grating with the optimal groove depth, the diffraction efficiencies are more uniform over the range of angles. Furthermore, according to Eq. (10), the overall diffraction efficiency for the optimal groove depth (Fig. 3) is 96%, whereas that for the other groove depth (Fig. 2) is only 93%.

We performed initial experiments to verify the above predictions for the diffraction efficiencies. The blazed reflection grating was recorded as a surface-relief grating with eight discrete binary steps by using multilevel lithographic techniques.[9] The grating was designed to have maximal efficiency for a wavelength of $\lambda_0 = 10.6$ μm (from a CO_2 laser) at a normal incidence ($\theta_i = 0$). Therefore the groove depth was chosen, according to Eq. (7), to be $d = 0.5\lambda_0 = 5.3$ μm. The efficiency of the first diffraction order η_1 for these parameters was measured to be 88%. This value is 7% lower than the theoretical maximum for an eight-level phase grating[9]; this reduction in efficiency is due to inaccuracies in the realization process. Then, η_1 was measured at several other angles of incidence. The results are shown in Fig. 4, together with the corresponding theoretical results from Fig. 2. The experimental diffraction efficiencies were normalized to 100% at normal incidence to make possible a quantitative comparison with the theoretical results. As is shown, the experimental reduction in diffraction efficiency is in close agreement with the calculated one.

Next, the wavelength of the CO_2 laser was changed to $\lambda_0 = 9.6$ μm, and the diffraction efficiency measurements were repeated. The maximal groove depth, still 5.3 μm, thus becomes $0.55\lambda_0$ (instead of $0.5\lambda_0$ in the previous experiment, where λ_0 was 10.6 μm). This corresponds to the optimal groove depth for the angular range of $-45° < \theta < 45°$. The results are presented in Fig. 5, together with the corresponding theoretical results from Fig. 3. Again, the maximal experimental efficiency was normalized to 100% (here at an angle of incidence of 15°). By comparing Figs. 4 and 5, it is evident that the experimental diffraction efficiencies are indeed more uniform over the range of angles for the grating with the optimal groove depth of $0.55\lambda_0$.

4. RANGE OF WAVELENGTHS

Let us now consider the relatively simple case in which the incoming light is oriented at a specific angle. We assume a normal incidence ($\theta_i = 0$) and a uniform distribution of the spectral range $\lambda_{\min} < \lambda < \lambda_{\max}$ (white light). These assumptions are expressed mathematically by the distribution function having the form

$$\omega(\theta_i, \lambda) = \begin{cases} \delta(\theta_i) & \text{if } \lambda_{\min} < \lambda < \lambda_{\max} \\ 0 & \text{elsewhere} \end{cases}. \tag{13}$$

The overall diffraction efficiency of the grating is obtained by incorporating Eq. (13) into Eq. (8) to yield

$$E(d) = \int_{\lambda_{\min}}^{\lambda_{\max}} \eta_1(\theta_i = 0, \lambda, d) \mathrm{d}\lambda. \tag{14}$$

Following the same procedure as in Section 3 [i.e., Eqs. (10)–(12)], we find the optimal groove depth d_{opt} that maximizes the overall diffraction efficiency to be

$$d_{\mathrm{opt}} = 0.5 \frac{\lambda_{\max}\lambda_{\min}}{\lambda_{\max} - \lambda_{\min}} \ln \frac{\lambda_{\max}}{\lambda_{\min}}. \tag{15}$$

It is interesting to compare this choice for the optimal groove depth with those that would give a diffraction efficiency of 100% for a specific wavelength within the range. In particular, we compare our choice with those where the

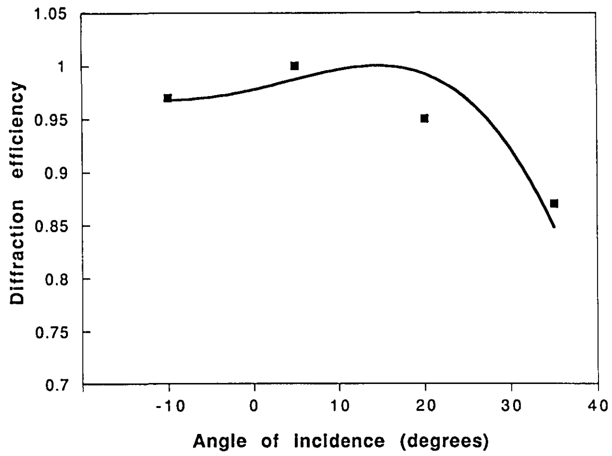

Fig. 5. Experimental and calculated diffraction efficiencies as a function of the angle of incidence for a blazed grating with a groove depth of 0.55λ (optimal depth). Curve: scalar calculation; squares: experimental results.

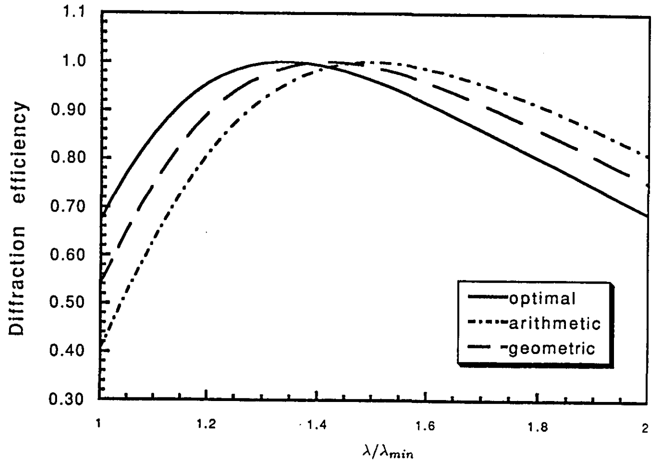

Fig. 6. Calculated diffraction efficiency as a function of the wavelength (in units of λ_{min}) for a blazed grating with groove depths chosen in accordance with geometric average, arithmetic average, and optimal calculations.

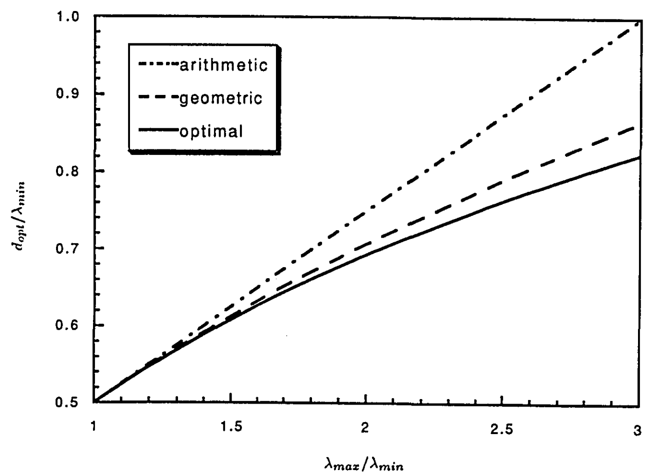

Fig. 7. Groove depths of the blazed grating (in units of λ_{min}) as a function of $\lambda_{max}/\lambda_{min}$ for the arithmetic average, geometric average, and optimally calculated wavelengths.

specific wavelength is an arithmetic average, $(\lambda_{min} + \lambda_{max})/2$, and where it is a geometric average $(\lambda_{min}\lambda_{max})^{1/2}$. We calculated the diffraction efficiencies and the optimal groove depths for each choice. The results are shown in Figs. 6 and 7. Figure 6 shows the diffraction efficiencies as a function of λ (in units of λ_{min}) for a wavelength range of $\lambda_{max}/\lambda_{min} = 2$. As is shown, the diffraction efficiency for the optimal groove depth is the most uniform over the range of wavelengths.

Figure 7 shows the optimal groove depth (in units of λ_{min}) as a function of the ratio $\lambda_{max}/\lambda_{min}$. As is evident, the groove depths derived in accordance with the geometric average are similar to the optimal depths. The groove depths derived in accordance with the arithmetic average are considerably different. It can therefore be concluded that the simple geometric average may be adequate for relatively moderate spectral ranges.

5. CONCLUSION

We have shown that it is possible to obtain a high diffraction efficiency with blazed diffraction gratings even when the incident light contains a broad range of wavelengths and arrives from a wide range of angles. The high efficiencies are achieved by optimizing the groove depth in the gratings. The gratings with the optimal groove depth also have improved uniformity in diffraction efficiency over the range of incidence angles and wavelengths, a property that may be advantageous in many applications.

Finally, our optimization approach can be readily generalized to include higher diffraction orders and thick gratings for which the period is of the same order of magnitude as the wavelengths. However, the scalar theory may no longer be valid, so the diffraction efficiencies must be calculated by the rigorous vectorial approach.[8] In such a rigorous approach the optimal groove depth could be found by numerical rather than by analytic techniques. Moreover, the small facet may no longer be vertical to the grating plane, as for the thin grating, so its angle must be optimized as well.

Throughout the paper we assumed that the groove shape is triangular (blazed) and considered the effects of its depth. It is possible that different groove shapes would result in higher diffraction efficiencies over a broad range of wavelengths and incidence angles. We considered binary, sinusoidal, and parabolic groove shapes, but these resulted in lower diffraction efficiencies than were obtained with the blazed gratings. Nevertheless, so far we have not been able to prove rigorously that the blazed groove shape is indeed optimal in this respect.

We conclude by noting that alternative approaches may be considered when one deals with blazed holographic gratings that are illuminated with nonlaser light. For example, it is possible to require maximal uniformity of the diffraction efficiency within the spectral and angular range, rather than maximal total efficiency as we required. This would lead to some modifications in the mathematical derivations, but the general formalism would be similar.

REFERENCES

1. R. Magnusson and T. K. Gaylord, "Diffraction efficiencies of thin phase gratings with arbitrary grating shape," J. Opt. Soc. Am. A **6**, 806–809 (1978).
2. E. G. Loewen, M. Nevière, and D. Maystre, "Grating efficiency theory as it applies to blazed and holographic gratings," Appl. Opt. **16**, 2711–2721 (1977).
3. E. G. Loewen, M. Nevière, and D. Maystre, "On an asymptotic theory of diffraction gratings used in the scalar domain," J. Opt. Soc. Am. **68**, 496–502 (1978).
4. G. J. Swanson and W. B. Velkamp, "Binary lenses for use at 10.6 micrometers," Opt. Eng. **24**, 791–795 (1985).
5. E. Hasman and A. A. Friesem, "Analytic optimization for holographic optical elements," J. Opt. Soc. Am. A **6**, 62–72 (1989).
6. D. Maystre, M. Nevière, and R. Petit, in *Electromagnetic Theory of Gratings,* R. Petit, ed. (Springer-Verlag, Berlin, 1980), p. 213.
7. E. K. Popov, L. V. Tsonev, and E. G. Loewen, "Scalar theory of transmission relief gratings," Opt. Commun. **80**, 307–311 (1991).
8. M. G. Moharam and T. K. Gaylord, "Rigorous coupled-wave analysis of metallic surface-relief gratings," J. Opt. Soc. Am. A **3**, 1780–1787 (1986).
9. E. Hasman, N. Davidson, and A. A. Friesem, "Efficient multilevel phase holograms for CO_2 lasers," Opt. Lett. **16**, 423–425 (1991).