Achromatic phase retarder by slanted illumination of a dielectric grating with period comparable with the wavelength

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We discuss some properties of dielectric gratings with period comparable with the illuminating wavelength for slanted illumination (this illumination geometry is often referred to as concical mounting). We demonstrate the usefulness of such an illuminating geometry. We show that the threshold period (under which only the zeroth transmission and reflection orders are nonevanescent) can be significantly higher, thereby easing fabrication constraints, and that this illumination setup makes it possible to design achromatic phase retarders. Such a design, for an achromatic quarter-wave plate with $\lambda/60$ uniformity of the retardation phase in the 0.47–0.63-μm wavelength interval, is demonstrated. © 2001 Optical Society of America

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1. Introduction

Phase retarders are devices that change the polarization state of a beam incident on them. This is achieved by use of the birefringent properties of phase retarders: The TE and the TM components of the incoming light undergo different phase shifts, so the relative phase difference between TE and TM polarizations (and hence the polarization state of the light) changes. Besides natural crystals, high-density dielectric gratings can also exhibit birefringence properties. This phenomenon called form birefringence[1] can actually be a much stronger effect than the natural birefringence of crystals.

Two main scalar theories have been developed to determine and analyze the diffraction from dielectric gratings. One is the effective medium theory (EMT),[1] which is suitable for gratings whose period Λ is much smaller than the illuminating wavelength λ. The other is the scalar diffraction theory,[2] which is suitable for gratings whose period Λ is much larger than the illuminating wavelength λ. Specifically, for the regime of $\Lambda \ll \lambda$ and perpendicular incidence, EMT accurately describes the intensities of the transmitted and the reflected zero-order diffractions for both TE and TM incoming polarizations, as well as the phase difference between the TE and the TM polarizations. Alternatively, for the opposite regime of $\Lambda \gg \lambda$, the scalar diffraction theory is sufficient for determining all the diffraction orders. However, in the intermediate regime, when $\Lambda \sim \lambda$, EMT and scalar diffraction become inaccurate. The second-order EMT,[3] which includes a second-order correction of the finite ratio Λ/λ, has somewhat improved accuracy for $\Lambda \leq \lambda/2$ but eventually also fails as Λ approaches λ. Thus in general it is necessary to resort to rigorous vectorial calculations.[4]

In this paper we investigate the polarization properties of dielectric gratings at the $\Lambda \sim \lambda$ regime and show how they can be effectively exploited for phase-retardation applications. Such gratings have several attractive features when used as phase retarders. First, their relatively low spatial resolution alleviates the fabrication problems of surface-relief dielectric gratings, which are of particular significance with visible and UV illuminations. Second, their effective birefringence is relatively strong—it can even be stronger than that of $\Lambda \ll \lambda$ gratings—thereby increasing the attainable phase

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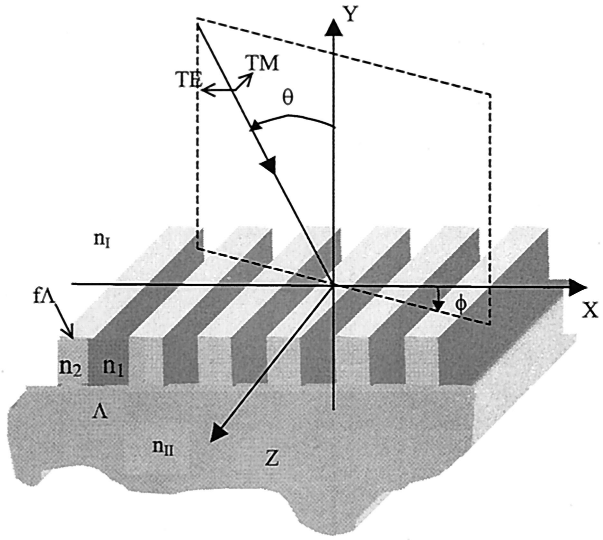

Fig. 1. Illuminating geometry of a dielectric grating.

retardation. Finally, their wavelength sensitivity is relatively low, so they can be exploited as a phase retarder over a broad range of wavelengths. However, such gratings can have higher diffraction orders, which can result in the reduction and strong variations of the intensity of transmitted zero-order diffraction. We show, however, that by means of optimizing the grating period and thickness of the grating, and the illumination geometry, these effects can largely be suppressed. For our investigations we use rigorous coupled-wave analysis (RCWA)[5] and the results from EMT for dielectric gratings in the $\Lambda \ll \lambda$ regime.

2. Basic Relations

For any dielectric grating and arbitrary incidence illumination geometry, as depicted in Fig. 1, the threshold period under which only the transmitted and the reflected zero-order diffractions are nonevanescent is[4]

$$\Lambda_{th} = \frac{\lambda}{n_I \sin\theta \cos\phi + (n_{II}^2 - n_I^2 \sin^2\theta \sin^2\phi)^{1/2}}, \tag{1}$$

where ϕ is the azimuth angle, θ is the angle of incidence; and n_I and n_{II} are the refractive indices of the superstrate and the substrate media, respectively. As shown in Fig. 1, the TE polarization is perpendicular to the plane of incidence and the TM polarization lies in the plane of incidence.

Equation (1) indicates that for $\phi = 90°$ and sufficiently large θ even gratings of $\Lambda > \lambda$ can behave like zero-order (or subwavelength) gratings; the threshold period Λ_{th} has its maximum at $\phi = 90°$, $\theta = 90°$. However, for large θ (typically for $\theta > 70°$), the efficiency of the zero-order transmitted wave is significantly reduced by Fresnel reflection at the interfaces. Hence a trade-off must be reached between the maximum angle of incidence (in terms of efficiency) and the minimum period (in terms of fabrication ease).

According to EMT, for binary dielectric gratings with $\Lambda \ll \lambda$ and normal incidence illumination (i.e., $\theta = 0°$), the effective refractive indices n_{TE} and n_{TM} for the TE and the TM polarizations are[4]

$$n_{TE} = [f n_2^2 + (1 - f) n_1^2]^{1/2}, \tag{2}$$

$$n_{TM} = [f n_2^{-2} + (1 - f) n_1^{-2}]^{-1/2}, \tag{3}$$

where f is the duty cycle of the grating and n_1 and n_2 are the refractive indices of the two materials composing the grating (with surface relief gratings $n_1 = n_I$ and $n_2 = n_{II}$). A binary $\Lambda \ll \lambda$ grating of thickness t can thereby be treated as a homogeneous birefringent layer of the same thickness. With the simple model of multiple reflections between the two boundaries of a homogeneous layer (similar to those of a Fabry–Perot resonator) and the Fresnel formulas for transmission and reflection coefficients at the superstrate–layer and layer–substrate interfaces, the phase difference between the transmitted TE and TM polarizations is found, for normal illumination, to be

$$\begin{aligned}\Delta\psi = \arg\Big[& n_{TE}(n_I + n_{II})\cos\Big(\frac{2\pi}{\lambda} n_{TE} t\Big) \\ &+ i(n_I n_{II} + n_{TE}^2)\sin\Big(\frac{2\pi}{\lambda} n_{TE} t\Big)\Big] \\ - \arg\Big[& n_{TM}(n_I + n_{II})\cos\Big(\frac{2\pi}{\lambda} n_{TM} t\Big) \\ &+ i(n_I n_{II} + n_{TM}^2)\sin\Big(\frac{2\pi}{\lambda} n_{TM} t\Big)\Big],\end{aligned} \tag{4}$$

where i is the imaginary unit.

Next, using Eq. (4), we calculated $\Delta\psi$ as a function of grating thickness t and compared the results with those from RCWA. The binary grating and illumination parameters had the following values: $\lambda = 0.6328\ \mu m$, $n_1 = n_I = 1$, and $n_2 = n_{II} = 1.64$ (the index of refraction of the Microposit S1800 photoresist series at $\lambda = 0.6328\ \mu m$), $f = 0.5$, and $\theta = 0°$ (ϕ is undefined). The results are presented in Fig. 2. These show $\Delta\psi$ as a function of grating thickness (calculated from EMT and RCWA) for three different values of Λ; the EMT does not depend on λ/Λ and is thus calculated only once. The results reveal an excellent agreement between EMT and RCWA for very small Λ (0.1λ). The results also show that a reasonable agreement is obtained for $\Lambda = 0.6\lambda$, which—although already in the $\Lambda \sim \lambda$ regime—is still a zero-order grating, according to Eq. (1). The oscillations around a linear line [which is expected from the simplified relation $\Delta\psi \propto (n_{TE} - n_{TM})t$] result from multiple reflection from the grating boundaries. Finally, the results for $\Lambda = 0.9\lambda$ show large deviations between RCWA and EMT, as are indeed expected, since in this case a first-order diffraction wave is allowed to propagate as a substrate mode.

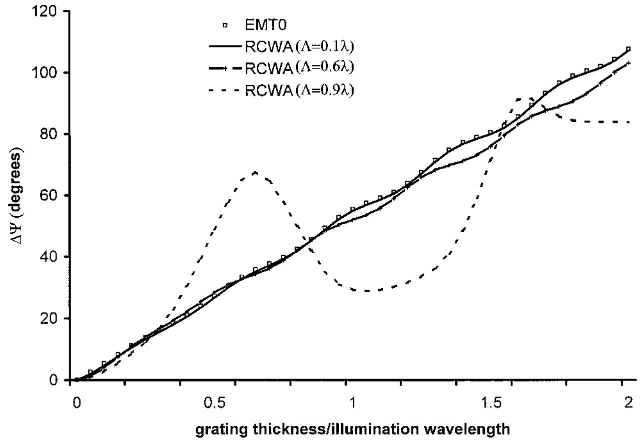

Fig. 2. Comparison between zero-order EMT and RCWA for gratings with different grating periods. $\Delta\psi$ as a function of grating thickness.

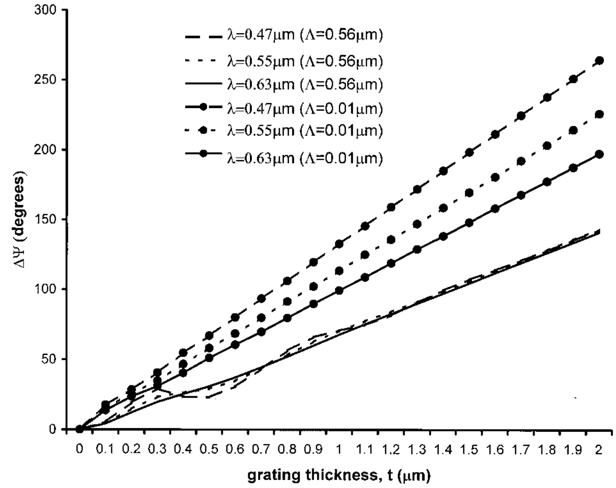

Fig. 3. $\Delta\psi$ as a function of grating thickness, for $\lambda = 0.47$, 0.55, and 0.67 μm and for different grating periods. The illumination parameters are $\phi = 90°$, $\theta = 60°$.

Several important features emerge from the results of Fig. 2. First, form birefringence still exists, with the same order of magnitude as in the $\Lambda \ll \lambda$ case. Moreover, for some parameters (e.g., $t \sim 0.65\lambda$), the phase difference between the zero orders of the TE and the TM polarizations is nearly twice as large as in the $\Lambda \ll \lambda$ regime, which can reduce the usually needed large grating depths and thereby ease fabrication. It must be noted, however, that in the $\Lambda \sim \lambda$ regime there may be a reduction of efficiency in the zero-order diffraction intensity, owing to the addition of higher diffraction orders. In this particular case, the throughput light efficiency for zero-order diffraction is only ~50% (both for TE and TM) at $t \sim 0.65\lambda$, and the deviations from linearity are quite large. We also calculated $\Delta\psi$ as a function of thickness, using the second-order EMT, whose results in terms of diffraction efficiency are shown to be in good agreement with RCWA even near the $\Lambda \sim \lambda$ regime.[6] We found, however, that in terms of $\Delta\psi$, the results of the second-order EMT significantly deviate from those of RCWA, just as those from the zero-order EMT do for $\Lambda = 0.9\lambda$.

3. Achromatic Phase Retarder

When a dielectric grating with $\Lambda \ll \lambda$ is illuminated with a plane wave at normal incidence, a simple expression to determine $\Delta\psi$ is

$$\Delta\psi(\lambda, t) = (2\pi/\lambda)(n_{\mathrm{TE}} - n_{\mathrm{TM}})t. \tag{5}$$

Equation (5) is only an approximation of Eq. (4), because it neglects multiple reflections inside the grating region. Nevertheless, this simplified expression can be helpful in providing an intuitive picture, even for gratings illuminated at arbitrary incidence angles and having periods comparable with the illuminating wavelength. The expression is particularly useful when we consider the results from higher-order EMT, which showed that $n_{\mathrm{TE}} - n_{\mathrm{TM}}$ is approximately linearly proportional to λ in the $\Lambda \sim \lambda$ regime.[4] This implies, according to Eq. (5), that $\Delta\psi$ could be independent of wavelength. Thus it may be possible to design a dielectric grating whose period is comparable with the wavelength ($\Lambda \sim \lambda$), which will behave as an achromatic phase retarder over a relatively broad range of wavelengths. In the design, care must be taken to ensure that the usual higher-order diffractions, in gratings with $\Lambda \sim \lambda$, are low so as to obtain a reasonable transmitted zero-order diffraction intensity.

When designing a dielectric grating for the achromatic phase retarder, one has several free parameters available: grating shape, t, n_1, n_2, n_{I}, n_{II}, Λ, and the illumination angles ϕ and θ. Some of these, however, are set by available grating recording materials and practical recording and illumination geometries. We found that the optimal illumination geometry is $\phi = 90°$ and $\theta \neq 0$, which, according to Eq. (1), yields a high Λ_{th}. With this geometry, even if $\Lambda > \Lambda_{\mathrm{th}}$, the efficiency of the inherent higher diffraction orders can be significantly lower than that of the transmitted zero-order diffraction. For the design procedure, first, the desired wavelength interval and $\Delta\psi$ have to be specified. Then, by iteratively changing the available free parameters, we can approximate the specifications to a high degree of precision.

To illustrate our approach, we designed and analyzed a phase retarder for the wavelength interval 0.47–0.63 μm and for $\Delta\psi = 90°$ (i.e., achromatic quarter-wave plate). We assumed that the grating shape is sinusoidal, with $n_1 = 1$, $n_2 = 1.64$. The free parameters in this case were t, Λ, and θ. With the iterative procedure we obtained the following values for these parameters: $t = 1.29$ μm, $\Lambda = 0.56$ μm, and $\theta = 60°$. We calculated the phase retardation $\Delta\psi$ as a function of grating thickness for such a phase retarder. The results, along with results for a phase retarder of a grating with $\Lambda \ll \lambda$, are shown in Fig. 3; in both cases the illumination geometry was $\theta = 60°$ and $\phi = 90°$. As expected, in accordance with Eq. (5), $\Delta\psi$ for the phase retarder with the grating of

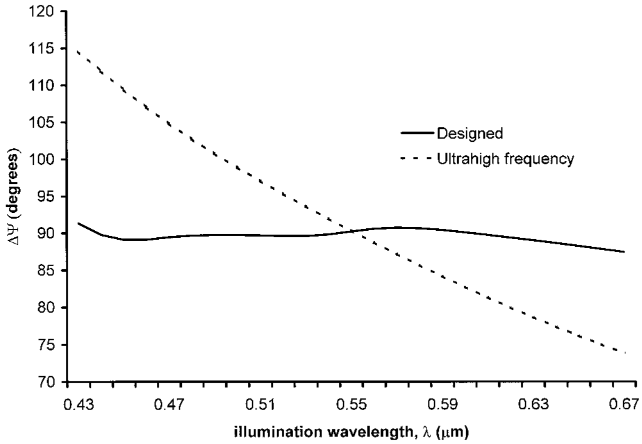

Fig. 4. Achromatic phase retarder. $\Delta\psi$ as a function of illumination wavelength.

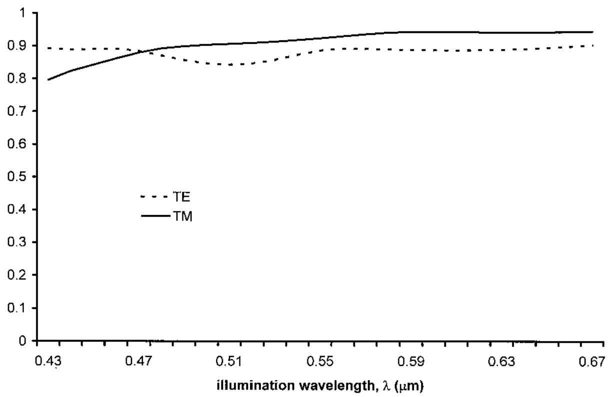

Fig. 5. Achromatic phase retarder. TE and TM efficiencies as a function of illumination wavelength.

$\Lambda \ll \lambda$ has different slopes for different wavelengths. However, for the phase retarder that was designed according to our procedure with a grating of $\Lambda \sim \lambda$ and the specific illumination geometry, $\Delta\psi$ as a function of grating thickness is essentially the same for all the wavelengths.

As indicated in Fig. 3, a desired phase retardation $\Delta\psi$ can be obtained by a proper choice of the grating thickness. For example, a quarter-wave plate of $\Delta\psi = 90°$ corresponds to $t = 1.29$ μm. We can obtain phase retarders with other constant phase-retardation values over the same wavelength interval simply by choosing other values for t.

Figure 4 shows the phase retardation $\Delta\psi$ as a function of wavelength, for a phase retarder designed according to our procedure. The grating thickness t was 1.29 μm. For the specified wavelength interval 0.47–0.63 μm the deviation of $\Delta\psi$ from 90° is ±0.85°. For comparison we also include the results for a phase retarder with a sinusoidal grating of the same parameters, except that $\Lambda = 0.01$ μm and $t = 0.79$ μm (so that $\Delta\psi = 90°$ at the middle of the wavelength interval). This element with $\Lambda \ll \lambda$ behaves in accordance with EMT, where $\Delta\psi$ as a function of wavelength is simply a hyperbola. The deviation of $\Delta\psi$ from 90° is approximately ±13°, much larger than for our designed phase retarder, where $\Lambda = 0.56$ μm.

Figure 5 shows the efficiency of the transmitted zero-order diffraction as a function of wavelength for both TE and TM polarizations. These results were obtained for the phase retarder designed according to our procedure and analyzed with RCWA. The average TE efficiency is 0.87 with a 6% uniformity over the specified wavelength interval, and the average TM efficiency is 0.91, with a 7% uniformity. The corresponding amplitude transmission coefficients are 0.935 for the TE polarization and 0.96 for the TM polarization, with uniformities of 2.8% and 3.5%, respectively. These results indicate that, despite the existence of additional diffraction orders, it is possible to obtain a fairly high and uniform efficiency in the zero order. The difference in average transmission coefficients for the TE and the TM polarizations does not affect the purity of the outcoming circular polarization, provided that the incoming linear polarization is slightly rotated from 45° to the principal axis of the quarter-wave plate (by ~0.8° in our case). However, the nonuniformity (as a function of wavelength) of the TE and the TM transmissions is translated into an additional error in the retardation accuracy if the element is used in white light. According to our calculations, the combined effect of nonuniformity in amplitude transmission and the deviation of $\Delta\psi$ from 90° together result in an analogous average retardation error of 5.9° for our element in the 0.47–0.63-μm wavelength range. This corresponds to $\lambda/60$ (and polarization energy purity of 99.9993%), which is close to the specification of $\lambda/100$ for a high-quality commercial achromatic wave plate.[7]

Note that when the achromatic phase retarder is used with one wavelength at a time (e.g., with a tunable laser) instead of a white-light source then the nonuniformity of the transmission coefficients can be compensated for as explained above, and the accuracy of the quarter-wave plate is again ±0.85°, which corresponds to $\pm\lambda/420$.

We also note that the idea of using gratings in conical mountings as polarization elements has already been proposed[8,9] but only for a single wavelength, not as achromatic elements.

4. Concluding Remarks

Given the large number of free parameters that are available in our design procedure, it is possible to obtain achromatic phase retarders both for the visible range and for wavelength regions outside the visible spectrum. For example, by means of increasing n_2 or t, phase retarders for IR-wavelength ranges can be designed. It must be noted, however, that the increase in n_2 usually results in higher losses, owing to higher Fresnel reflections at the interfaces, and the increase in t often makes fabrication more difficult. Yet, because our phase retarders use gratings in the $\Lambda \sim \lambda$ regime, the fabrication requirements are generally much less stringent in terms of aspect ratio

($f\Lambda/t$) than in the case of phase retarders using gratings in the $\Lambda \ll \lambda$ regime. Moreover, the use of gratings with $\Lambda \sim \lambda$ may lead to the reduction of the angular sensitivity of phase retarders and possibly to reduce the wavelength and angular sensitivity of antireflection coatings.

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